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Wave intensity amplification and attenuation in non-linear flow: Implications for the calculation of local reflection coefficients

Jonathan Mynard [a,b,*], Daniel J. Penny [a,b,c], Joseph J. Smolich [a,b]

[a] Heart Research Group, Murdoch Children's Heart Research Institute, Flemington Road, Parkville, Victoria 3052, Australia
[b] Department of Paediatrics, University of Melbourne, Flemington Road, Parkville, Victoria 3052, Australia
[c] Department of Cardiology, Royal Children's Hospital, Flemington Road, Parkville, Victoria 3052, Australia

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A B S T R A C T

Local reflection coefficients (R) provide important insights into the influence of wave reflection on vascular haemodynamics. Using the relatively new time-domain method of wave intensity analysis, R has been calculated as the ratio of the peak intensities (R_{PI}) or areas (R_{CI}) of incident and reflected waves, or as the ratio of the changes in pressure caused by these waves ($R_{\Delta P}$). While these methods have not yet been compared, it is likely that elastic non-linearities present in large arteries will lead to changes in the size of waves as they propagate and thus errors in the calculation of R_{PI} and R_{CI}. To test this proposition, R_{PI}, R_{CI} and $R_{\Delta P}$ were calculated in a non-linear computer model of a single vessel with various degrees of elastic non-linearity, determined by wave speed and pulse amplitude (ΔP_+), and a terminal admittance to produce reflections. Results obtained from this model demonstrated that under linear flow conditions (i.e. as $\Delta P_+ \to 0$), $R_{\Delta P}$ is equivalent to the square-root of R_{PI} and R_{CI} (denoted by R^p_{PI} and R^p_{CI}). However for non-linear flow, pressure-increasing (compression) waves undergo amplification while pressure-reducing (expansion) waves undergo attenuation as they propagate. Consequently, significant errors related to the degree of elastic non-linearity arise in R_{PI} and R_{CI}, and also R^p_{PI} and R^p_{CI}, with greater errors associated with larger reflections. Conversely, $R_{\Delta P}$ is unaffected by the degree of non-linearity and is thus more accurate than R_{PI} and R_{CI}.

1. Introduction

It has long been recognised that the pressure and flow waves generated during ventricular systole are partially reflected from the vasculature and that these reflections make a significant contribution to ventricular afterload and overall haemodynamics (Brin and Yin, 1984; O'Rourke and Kelly, 1993; Duan and Zamir, 1995; Koh et al., 1998; Penny et al., 2008). Moreover, accurate quantitation of the degree of wave reflection is important in view of the increasing use of indices based on this measure in the management of clinical conditions such as hypertension (Nichols et al., 2008; Weber et al., 2007). Wave intensity analysis (WIA) is a relatively new time-domain method for investigating wave propagation and reflection in the circulation (Parker and Jones, 1990; Jones et al., 2002). Wave intensity (WI) is defined as the product of the rates of change of pressure (P) and velocity (U), and thus a change in P and U is always associated with a 'wave'. With knowledge of wave speed, the net WI profile can be separated into forward components arising from the heart and backward components originating from the circulation (Parker and Jones, 1990). Forward- and backward-travelling waves can be further classified as either 'compression waves' which cause pressure to increase or 'expansion waves' which cause pressure to decrease. Under normal conditions, the initial ventricular impulse produces a large forward compression wave (FCW) in early systole, which accelerates blood and increases pressure. Subsequently, a forward expansion wave (FEW) generated in late-systole reduces pressure and flow before valve closure (Parker et al., 1988; Penny et al., 2008). The FCW is often followed by a smaller backward compression (BCW) and/or a backward expansion wave (BEW), which arise from downstream reflection of FCW (Hollander et al., 2001; Khir et al., 2001; Khir and Parker, 2005; Zambanini et al., 2005; Penny et al., 2008).

The amount of wave reflection from a given reflection site can be quantified by calculation of a local reflection coefficient (R) (Latham et al., 1985; Greenwald et al., 1990; Khir and Parker, 2002; Segers et al., 2006). Three approaches have been employed for

* Corresponding author at: Department of Cardiology, Royal Children's Hospital, Flemington Road, Parkville, Victoria 3052, Australia. Tel.: +61 3 9345 5922; fax: +61 3 9345 6001.
E-mail address: jonathan.mynard@mcri.edu.au (J. Mynard).

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calculating R using WIA. The first uses the ratio of changes in pressure related to the forward (ΔP_+) and backward (ΔP_-) waves (Khir and Parker, 2002), so that

$$R_{\Delta P} = \frac{\Delta P_-}{\Delta P_+} \tag{1}$$

With the remaining approaches, R is calculated directly from the wave magnitude, which can be quantified by peak WI (Jones et al., 1992, 2002; Bleasdale et al., 2003; Fujimoto et al., 2004; Khir and Parker, 2005; Penny et al., 2008; Smolich et al., 2008) or the wave area (termed 'cumulative intensity') (Davies et al., 2006; Penny et al., 2008; Smolich et al., 2008). Thus, R has been calculated from the ratio of the cumulative intensities of backward (CI_-) and forward (CI_+) waves (Hollander et al., 2001; Hobson et al., 2007; Penny et al., 2008; Smolich et al., 2008):

$$R_{\mathrm{CI}} = \pm\frac{|\mathrm{CT}_-|}{\mathrm{CI}_+} \tag{2}$$

Alternatively, R has been obtained from the peak wave intensities of backward (PI_-) and forward waves (PI_+) (Bleasdale et al., 2003) as follows:

$$R_{\mathrm{PI}} = \pm\frac{|\mathrm{PI}_-|}{\mathrm{PI}_+} \tag{3}$$

Note that the sign of both WI-derived reflection coefficients (R_{PI} and R_{CI}) is positive if the reflected wave is the same type as the incident wave (compression or expansion) and negative if they are different.

These methods for calculating R have never been compared, and it is not known whether they are equivalent. In addition, consideration of vascular properties suggests that calculation of R directly from WI profiles may itself be subject to error. Specifically, it is well-established that flow in large arteries, where WIA is usually performed, is non-linear due primarily to the pressure-dependant compliance of the arterial wall (Bodley, 1971; Raines et al., 1974; Stergiopulos et al., 1993; Mynard and Nithiarasu, 2008). These elastic non-linearities cause the early systolic rise in arterial pressure to *steepen* as the pulse propagates distally. Since WI is dependent on the rate of change of P and U, it would be expected that this steepening would alter the size of associated waves (Jones et al., 1992). However, any such alterations in wave magnitude during non-linear propagation between a measurement site and a reflection site will, in turn, affect the value of R derived from WI waveforms.

Accordingly, the aim of this study was to investigate the reliability of calculating R using the three available WIA approaches under various degrees of non-linearity. To achieve this, WIA was applied to a non-linear one-dimensional computer model of a single vessel, with the degree of elastic non-linearity varied by simulating a range of physiological input pulse amplitudes, wave speeds and vessel cross-sectional areas.

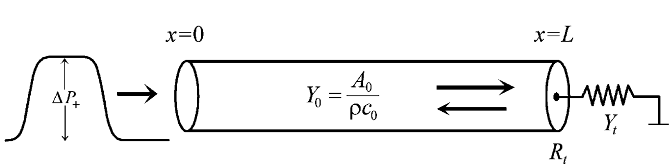

Fig. 1. The single segment 1D model has a length L and characteristic admittance Y_0. A forward pressure is prescribed at the inlet. At the outlet, a terminal admittance Y_t results in wave reflection when $Y_0 \neq Y_t$ with a reflection coefficient R_t.

2. Methods

2.1. Computer model

The model (Fig. 1) consisted of a single vessel of length $L = 25$ cm with uniform initial cross-sectional area (A_0) and wave speed (c_0), where the subscript indicates values at zero transmural pressure. The vessel terminates in an admittance (Y_t) that produces a reflection coefficient R_t, according to

$$R_t = \frac{Y_0 - Y_t}{Y_0 + Y_t} \tag{4}$$

with the characteristic admittance of the vessel defined as

$$Y_0 = \frac{1}{Z_0} = \frac{A_0}{\rho c_0} \tag{5}$$

where Z_0 is characteristic impedance and ρ is blood density (1.06 g/cm^3). Reflection coefficients have limiting values of 1 for a complete positive reflection (when $Y_t \ll Y_0$ due to a total occlusion), −1 for a complete negative reflection (when $Y_t \gg Y_0$ in the case of an opening into an infinite reservoir), and zero when no reflection is present ($Y_t = Y_0$, the 'well-matched' case).

Full details of the computer model have been previously described (Mynard and Nithiarasu, 2008) and are briefly provided in the Appendix. The input was a forward component of pressure composed of two sigmoid curves (Mynard and Nithiarasu, 2008) with an amplitude of ΔP_+ (Fig. 1) while a known reflection coefficient R_t was prescribed at the outlet. WIA was applied to the model results using P and U obtained from $x = 0$ unless otherwise stated.

Note that for linear flow, $A(t)$ and $c(t)$ are approximately constant and $U \ll c$, but for non-linear flow, these approximations break down. Thus, while all simulations were performed with the non-linear governing equations, quasi-linear flow was assessed by employing very small ΔP_+ (0.075 mmHg), in which case $c(t)$ and $A(t)$ were effectively constant and $U \ll c$.

2.2. Wave intensity analysis

Wave intensity represents the summation of infinitesimal wavelets and was initially defined (Parker et al., 1988; Parker and Jones, 1990) as $\mathrm{d}P\,\mathrm{d}U$, where $\mathrm{d}P$ and $\mathrm{d}U$ are incremental changes in P and U over one sample interval; however time-corrected wave intensity $\mathrm{WI} = (\mathrm{d}P/\mathrm{d}t)(\mathrm{d}U/\mathrm{d}t)$, employed in this study, is increasingly being used (Jones et al., 2002; Niki et al., 2002; Penny et al., 2008; Smolich et al., 2008) since it is independent of sample rate.

By convention, all forward waves (WI_+) in WIA are positive and all backward waves (WI_-) are negative. The forward (+) and backward (−) components of WI (Jones et al., 2002) are defined as

$$\mathrm{WI}_+ = \frac{\mathrm{d}P_+}{\mathrm{d}t}\frac{\mathrm{d}U_+}{\mathrm{d}t} \quad \text{and} \quad \mathrm{WI}_- = \frac{\mathrm{d}P_-}{\mathrm{d}t}\frac{\mathrm{d}U_-}{\mathrm{d}t} \tag{6}$$

In these equations, $\mathrm{d}P_\pm/\mathrm{d}t$ and $\mathrm{d}U_\pm/\mathrm{d}t$ are the forward and backward components of $\mathrm{d}P/\mathrm{d}t$ and $\mathrm{d}U/\mathrm{d}t$, which are calculated via the water hammer principle (Parker and Jones, 1990) using wave speed $\bar{c}$ (the overbar denoting a constant, average value) and ρ:

$$\frac{\mathrm{d}P_\pm}{\mathrm{d}t} = \frac{1}{2}\left(\frac{\mathrm{d}P}{\mathrm{d}t} \pm \rho\bar{c}\frac{\mathrm{d}U}{\mathrm{d}t}\right) \quad \text{and} \quad \frac{\mathrm{d}U_\pm}{\mathrm{d}t} = \frac{1}{2}\left(\frac{\mathrm{d}U}{\mathrm{d}t} \pm \frac{1}{\rho\bar{c}}\frac{\mathrm{d}P}{\mathrm{d}t}\right) \tag{7}$$

Note that when applying WIA to the model results, we have *not* used $c_0 = \bar{c}$, but rather mean $c(t)$ (see Eq. (A.5) in the Appendix).

Integration of Eq. (7) yields forward and backward components of pressure and velocity (Westerhof et al., 1972).

$$P_\pm = \frac{1}{2}[(P - P^0) \pm \rho\bar{c}(U - U^0)] \quad \text{and} \quad U_\pm = \frac{1}{2}[(U - U^0) \pm \frac{1}{\rho\bar{c}}(P - P^0)] \tag{8}$$

where P^0 and U^0 are the starting values. Note that ΔP_+ and ΔP_- in Eq. (1) refer to changes in P_+ and P_-, respectively. Measured (net) P and U are equal to the sum of forward and backward components and the starting values, however in this study, $P^0 = U^0 = 0$.

Net wave intensity ($\mathrm{WI} = \mathrm{WI}_+ + \mathrm{WI}_-$) does not depend on wave speed and involves no linearising assumptions. However, calculation of $\mathrm{WI}_\pm$, $P_\pm$ and $U_\pm$ assumes constant wave speed and that these components add linearly.

2.3. Calculation of reflection coefficients

Reflection coefficients were calculated with Eqs. (1)–(3) using the measurements shown in Fig. 2.