Journal of Biomechanics 41 (2008) 2659–2666

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

On modelling nonlinear viscoelastic effects in ligaments

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ARTICLE INFO

Article history:
Accepted 10 June 2008

Keywords:
Large deformation
Nonlinear viscoelasticity
Hyperelasticity
Anisotropy
Ligaments
Finite element analysis

ABSTRACT

Experiments in human ligaments revealed that the rate of stress relaxation in such materials is strain dependent. This nonlinear behavior requires therefore a modified description of the standard quasilinear viscoelasticity theory commonly used in tissue biomechanics. The goal of this study is to characterize and demonstrate the importance of the nonlinear *stress-relaxation behavior* of ligaments undergoing finite deformation. The structural model presented herein is based on a local additive decomposition of the stress tensor into initial and non-equilibrium parts as resulted from the assumed structure of the free energy density function that generalizes Kelvin–Voigt nonlinear viscous models. We consider different viscoelastic behavior for the matrix and the fibers and the need of considering the strain dependency of this effect is clearly demonstrated.

Model parameters were fit to experimental data obtained in specimens undergoing finite deformation in two directions: longitudinal and transversal with respect to the directions of the collagen fibers. The model was then tested against several multi-axial loading situations. The strain dependent relaxation and the strain rate dependent behavior of the human medial collateral ligament were accurately predicted.

1. Introduction

The stretch and time-dependent behavior of biological tissues has been widely investigated by means of experimental tests. For example, Hingorani et al. (2004) and Bonifasi-Lista et al. (2005) found that ligaments exhibit a clear nonlinear viscoelastic response. They found that the creep rate depends on the applied stress and that the relaxation rate depend on the applied stretch. Silver et al. (2003) found a rate dependent mechanical behavior of the porcine aorta, vena cava and carotid artery. Other authors found time-dependent material behavior of blood vessels (Humphrey, 1995), cornea (Pinsky and Datye, 1991), brainstem (Ning et al., 2006), aortic valves (Grashow et al., 2006), pericardium (Sacks, 2000) and articular cartilage (Hayes and Mockros, 1971).

A full description of the mechanical response of soft biological tissue requires therefore including its nonlinear viscoelastic behavior. Considering therefore that the viscoelastic response is strain dependent in the low strain range and considering that the physiological loads on ligaments induce mostly low strains, a nonlinear viscoelastic model may gain a great relevance (Vena et al., 2006). For example, a strain dependent viscoelastic constitutive model is essential to simulate the complex loading conditions occurring in clinical application, such as the evolution along time of the initial prestress in bone–patellar tendon–bone grafts (Kampen et al., 1998).

Many viscoelastic constitutive models have been proposed to model biological soft tissues. A theory of quasilinear viscoelasticity was early proposed by Fung and is still widely used in the field of biomechanics (Fung, 1993). This model has, however, an important drawback: the information must be saved at every previous time step. Puso and Weiss (1998) formulated a time discretization algorithm of the convolution integral in which the relaxation function and the elastic constitutive behavior were split by means of a multiplicative decomposition, thus reducing the nonlinear response of the tissue to the latter. Other papers like those by Pioletti et al. (1998), Merodio and Goicolea (2007) and Merodio and Rajagopal (2007) modelled the isotropic and transversely isotropic viscohyperelastic behavior of ligaments by defining a viscous potential which involved 10 and 15 invariants, respectively. Other approaches for viscoelasticity are due to Johnson et al. (1996) and Vena et al. (2006) among others. There, the elastic response of the tissue and the time-dependent properties are independently modelled and combined into a convolution time integral. Provenzano et al. (2002) used a more

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doi:10.1016/j.jbiomech.2008.06.019

general description using the nonlinear theory of Schapery or the modified superposition method (Schapery, 1969). For a complete revision of nonlinear viscoelastic models see Drapaca et al. (2007) and references therein.

A more recent strategy to build a viscoelastic model writes the Helmholtz free energy density function as the sum of a hyperelastic term and a viscous term (Kaliske, 2000; Holzapfel et al., 2000; Natali et al., 2004, 2008; Peña et al., 2007a). The stress then expresses as the sum of an elastic component and a dissipative component, this latter defined in terms of several internal variables leading to generalized Maxwell (Holzapfel et al., 2000; Kaliske, 2000) or Kelvin–Voigt (Peña et al., 2007a) models for the viscous component. The main advantage of those models is the easy and efficient implementation of the resulting finite strain formulation and the associated algorithmic discretization into a finite element code. In addition, the models in Holzapfel et al. (2000) and Peña et al. (2007a) consider different viscoelastic behaviors for the matrix and the different families of fibers. In all cases the assumed dissipation is controlled by a set of linear differential equations, so the evolution equations obtained are linear, discarding therefore the stretch-dependence of the relaxation rate (Hingorani et al., 2004).

The objective of this paper is to present a more general viscoelastic model for the simulation of the response of ligaments. In particular, we have modified the fully 3D finite strain anisotropic viscohyperelastic Kelvin–Voigt model presented in Peña et al. (2007a) to predict the creep rate dependence on the applied stress and the relaxation rate dependence on the applied strain. We propose modified evolution equations that consider reduced relaxation and time functions that are strain dependent and different viscoelastic parameters for the matrix and the fibers.

2. Model formulation

Based on well-known experimental results previously published (Quapp and Weiss, 1998; Bonifasi-Lista et al., 2005; Woo and Young, 1991), the ligaments were assumed to be anisotropic hyperelastic materials. Since most biological soft tissues exhibit time-dependent behavior, an anisotropic viscoelastic model was developed to describe the here assumed nonlinear viscoelastic response of ligaments under large deformation (Peña et al., 2007b).

2.1. Anisotropic hyperelastic response of soft tissues

An usual way to formulate the elastic constitutive response of fibered soft tissues is to postulate the existence of a strain energy density function that depends on the direction of each family of fibers at a point $\mathbf{X}$ (two families were here considered with their directions defined by the unit vector fields $\mathbf{m}_0$ and $\mathbf{n}_0$, Peña et al., 2007b). The fiber moves with the material points of the continuum body, so the stretches λ_m and λ_n of the fibers defined as the ratio between their lengths at the deformed and reference configurations can be expressed as

$$\lambda_m^2 = \mathbf{m}_0 \cdot \mathbf{C}\mathbf{m}_0, \quad \lambda_n^2 = \mathbf{n}_0 \cdot \mathbf{C}\mathbf{n}_0 \tag{1}$$

where $\mathbf{m} = \mathbf{F}\mathbf{m}_0$ and $\mathbf{n} = \mathbf{F}\mathbf{n}_0$ are the unit vector of the fibers in the deformed configuration, $\mathbf{F} = \mathrm{d}\mathbf{x}/\mathrm{d}\mathbf{X}$ and $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ are the standard deformation gradient and the corresponding right Cauchy–Green strain measure.

To characterize isothermal processes, we postulate the existence of a unique decoupled representation of the strain energy density function Ψ (Simo and Taylor, 1991) that explicitly depends on both the right Cauchy–Green tensor $\mathbf{C}$ and the fiber directions $\mathbf{m}_0$ and $\mathbf{n}_0$ as (Spencer, 1954)

$$\begin{aligned}\Psi(\mathbf{C}, \mathbf{m}_0, \mathbf{n}_0) &= \Psi_{\mathrm{vol}}(J) + \bar{\Psi}(\bar{\mathbf{C}}, \mathbf{M}, \mathbf{N}) \\ &= \Psi_{\mathrm{vol}}(J) + \bar{\Psi}(\bar{I}_1, \bar{I}_2, \bar{I}_4, \bar{I}_6)\end{aligned} \tag{2}$$

where $\Psi_{\mathrm{vol}}(J)$ and $\bar{\Psi}$ are given scalar-valued functions of J, $\bar{\mathbf{C}}$, $\mathbf{M} = \mathbf{m}_0 \otimes \mathbf{m}_0$ and $\mathbf{N} = \mathbf{n}_0 \otimes \mathbf{n}_0$, respectively, that describe the volumetric and isochoric responses of the material (Holzapfel, 2000), $\bar{I}_1$ and $\bar{I}_2$ are the first two modified strain invariants of the symmetric modified Cauchy–Green tensor $\bar{\mathbf{C}}$ ($\mathbf{F} = J^{1/3}\bar{\mathbf{F}}$ and $\mathbf{C} = J^{2/3}\bar{\mathbf{C}}$). Finally, the pseudo-invariants $\bar{I}_4, \bar{I}_6$ characterize the kinematic response of the fibers (Spencer, 1954)

$$\bar{I}_4 = \mathbf{C} : \mathbf{M} = \lambda_m^2, \quad \bar{I}_6 = \mathbf{C} : \mathbf{N} = \lambda_n^2 \tag{3}$$

Note that while the invariants I_4 and I_6 have a clear physical meaning, the square of the stretch λ in the fibers directions, the influence of other invariants (I_5, I_7 and I_8, see Holzapfel, 2000) is difficult to evaluate due to the high correlation between them. For this reason and the lack of sufficient experimental data it is usual not to include these invariants in the definition of Ψ.

From the Clausius–Planck inequality, we obtain the constitutive equation for compressible hyperelastic materials as

$$\mathbf{S} = 2\frac{\partial \Psi(\mathbf{C}, \mathbf{M})}{\partial \mathbf{C}} = \mathbf{S}_{\mathrm{vol}} + \bar{\mathbf{S}} = Jp\mathbf{C}^{-1} + \bar{\mathbf{S}} \tag{4}$$

where the second Piola–Kirchhoff stress $\mathbf{S}$ consists of a purely volumetric contribution $\mathbf{S}_{\mathrm{vol}}$ and a purely isochoric one $\bar{\mathbf{S}}$ and p is the hydrostatic pressure.

The Cauchy stress tensor $\boldsymbol{\sigma}$ is $1/J$ times the push-forward of $\mathbf{S}$ ($\boldsymbol{\sigma} = J^{-1}\boldsymbol{\chi}_*(\boldsymbol{S})$) and is written as (Simo and Hughes, 1998)

$$\begin{aligned}\boldsymbol{\sigma} = p\mathbf{1} + \frac{2}{J}\Bigg[&\left(\frac{\partial \bar{\Psi}}{\partial \bar{I}_1} + \bar{I}_1\frac{\partial \bar{\Psi}}{\partial \bar{I}_2}\right)\bar{\mathbf{b}} - \frac{\partial \bar{\Psi}}{\partial \bar{I}_2}\bar{\mathbf{b}}^2 \\ &+ \bar{I}_4\frac{\partial \bar{\Psi}}{\partial \bar{I}_4}\mathbf{m} \otimes \mathbf{m} + \bar{I}_6\frac{\partial \bar{\Psi}}{\partial \bar{I}_6}\mathbf{n} \otimes \mathbf{n} \\ &- \frac{1}{3}\left(\frac{\partial \bar{\Psi}}{\partial \bar{I}_1}\bar{I}_1 + 2\frac{\partial \bar{\Psi}}{\partial \bar{I}_2}\bar{I}_2 + \frac{\partial \bar{\Psi}}{\partial \bar{I}_4}\bar{I}_4 + \frac{\partial \bar{\Psi}}{\partial \bar{I}_6}\bar{I}_6\right)\mathbf{1}\Bigg]\end{aligned} \tag{5}$$

with $\mathbf{I}$ the second-order identity tensor, and $\mathbf{b} = \bar{\mathbf{F}}\bar{\mathbf{F}}^{\mathrm{T}}$ and $\bar{\mathbf{b}} = J^{-2/3}\mathbf{b}$ the left and modified left Cauchy–Green tensors, respectively.

The associated decoupled elasticity tensor may be written as (Holzapfel, 2000)

$$\mathbb{C} = \mathbb{C}_{\mathrm{vol}} + \mathbb{C}_{\mathrm{iso}} = 2\frac{\partial \mathbf{S}_{\mathrm{vol}}}{\partial \mathbf{C}} + 2\frac{\partial \mathbf{S}_{\mathrm{iso}}}{\partial \mathbf{C}} \tag{6}$$

The complete expression of the elasticity tensor (6) in material and spatial descriptions is included for instance in Holzapfel (2000).

2.2. Time-dependent response under large deformation

In order to describe the viscoelastic effect we consider the finite strain anisotropic viscoelastic constitutive model proposed in Peña et al. (2007a). The concept of internal variable (Simo, 1987) is here applied, postulating the existence of an uncoupled free energy density function $\Psi(\mathbf{C}, \mathbf{Q})$ of the form

$$\begin{aligned}\Psi(\mathbf{C}, \mathbf{M}, \mathbf{Q}_{ij}) = \Psi^0_{\mathrm{vol}}(J) + \bar{\Psi}^0 - \frac{1}{2}\sum_{i=1}^{n}\sum_{k=m,f_1,f_2}(\bar{\mathbf{C}} : \mathbf{Q}_{ik}) \\ + \Xi\left(\sum_{i=1}^{n}\sum_{k=m,f_1,f_2}\mathbf{Q}_{ik}\right)\end{aligned} \tag{7}$$

where $\mathbf{Q}_{ik}$ may be interpreted as non-equilibrium stresses, in the sense of non-equilibrium thermodynamics, and remain unaltered under superposed spatial rigid body motions. $\mathbf{Q}_{im}$ are the isotropic contribution due to the matrix material associated therefore to I_1