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Communication

Development of a sliding mode control model for quiet upright stance

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ABSTRACT

Human upright stance appears maintained or controlled intermittently, through some combination of passive and active ankle torques, respectively representing intrinsic and contractile contributions of the ankle musculature. Several intermittent postural control models have been proposed, though it has been challenging to accurately represent actual kinematics and kinetics and to separately estimate passive and active ankle torque components. Here, a simplified single-segment, 2D (sagittal plane) sliding mode control model was developed for application to track kinematics and kinetics during upright stance. The model was implemented and evaluated using previous experimental data consisting of whole body angular kinematics and ankle torques. Tracking errors for the whole-body center-of-mass (COM) angle and angular velocity, as well as ankle torque, were all within ~10% of experimental values, though tracking performance for COM angular acceleration was substantially poorer. The model also enabled separate estimates of the contributions of passive and active ankle torques, with overall contributions estimated here to be 96% and 4% of the total ankle torque, respectively. Such a model may have future utility in understanding human postural control, though additional work is needed, such as expanding the model to multiple segments and to three dimensions.

1. Introduction

Bipedal upright stance is inherently unstable, requiring appropriate sensory integration and motor responses (joint torques) for maintenance [1]. While some work has assumed continuous control [2–5], recent experiments suggest that the underlying control mechanism is intermittent. For example, the plantarflexor muscles are not continuously active, but instead activated about three times per unidirectional sway [6]. In addition, the ankle plantarflexor/dorsiflexor muscles contract during sway away from the equilibrium position, but lengthen when the body sways back to the equilibrium position [7]. This intermittent muscle activation yields a "drop-catch" and "throw-catch" pattern of sway motion during upright stance, or ballistic intermittent motion [8,9].

Several intermittent controllers of upright stance have been developed, such as proportional-derivative (PD), bang-bang, and open-loop trajectory control [10–12]. Yet, accurate tracking of whole-body center-of-mass (COM) kinematics (e.g., angle, angular velocity, and angular acceleration) and ankle torque remains challenging. In particular, passive and active ankle torques during quiet upright stance, respectively representing the intrinsic and contractile contributions of the ankle musculature [13,14], are challenging to directly measure, yet these torque components make important contributions to movement stability in exercise and rehabilitation [15,16]. Sliding mode control is proficient at linear and nonlinear system dynamics tracking [17,18], and has been used in a variety of studies designed for human motion synthesis [19,20] and simulation [21].

Here, we developed a sliding mode control model, which is an intermittent controller in the aspect of being discontinuous, and assessed whether it can track COM kinematics and ankle torque during quiet upright stance. It was also assessed whether this controller yields modeled passive/active torques similar to those reported earlier using other methods, and which can provide additional information addressing whether quiet upright stance is primarily passively or actively controlled [2,22–26].

2. Methods

2.1. Experimental procedures and initial data processing

Data from an earlier experiment [27] were used here, obtained from a gender-balanced group of 32 healthy adults. As described in the noted study, participants completed three trials of quiet upright stance. Similar to previous studies [11,28], participants were instructed to stand as still as possible with their feet together, arms by their sides, head upright, and eyes closed (to induce larger postural sway [25]). In each 75-s trial, joint positions were estimated from reflective surface markers sampled at 20 Hz. Ground reaction forces (GRF) were obtained (at 100 Hz) from a force platform. Both GRF and joint kinematics were low-pass filtered (Butterworth, 5 Hz cutoff frequency, 4th order, zero lag). The sagittal plane location of the

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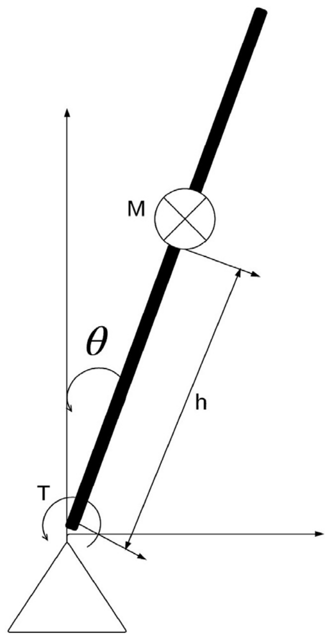

Fig. 1. Representation of single inverted pendulum model of the human body, where *h* is the distance between the whole-body center-of-mass (COM) and the ankle joint, ***θ*** is the angle of the COM from vertical, *M* is body mass, and *T* is ankle torque.

ankle was approximated using surface makers over both ankles, and averaged bilaterally. The location of the whole-body center-of-mass (COM) was assessed using a 7-link model (i.e., foot, shank, upper leg, trunk, head, lower arm, and upper arm), as in previous work [29]. Ankle torque in the sagittal plane was estimated from force platform data using a simple biomechanical model that included the foot/ankle and GRF [30].

2.2. Sliding mode controller

Given the limited inter-joint motion in quiet upright stance, the human body is often simplified to a single-segment inverted pendulum [11,22]. As such, in this initial investigation a 2D (sagittal plane), single-segment, inverted pendulum model of the body was employed (Fig. 1).

The plant dynamics for this model is

$$I\ddot{\theta} \;-\; Mghsin(\theta) = T \tag{1}$$

where I is the moment of inertia of the human body (rotation about the ankle joint), θ is the whole body COM angle (sagittal plane, relative to vertical), $\ddot{\theta}$ represents whole body COM angular acceleration, M indicates body mass, g is the gravitational constant, h indicates the distance between the COM and ankle joint, and T represents the plantar/dorsiflexion torque generated by the ankle.

Sliding mode control starts by formulating a control error, as

$$\tilde{q} = q - \; q_d = \begin{bmatrix} \tilde{q} & \dot{\tilde{q}} & \cdots & \tilde{q}^{(n-1)} \end{bmatrix}^T \tag{2}$$

where q is the state variable (here, predicted COM angle θ), and q_d is the desired state (actual COM angle). We chose n = 2, forming proportional and first-order derivative control components (PD controller), and constructed the sliding surface as

$$s = \dot{\tilde{q}} + \lambda\tilde{q} \tag{3}$$

where λ is a positive numeric value that affects the rate of estimated state convergence to the desired state [31].

Based on Lyapunov stability conditions [31], the sliding surface needs to satisfy

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{4}$$

where η is strictly positive. This specifies that the control trajectory points toward the sliding surface. Sliding mode control forces the initial trajectory to converge from an arbitrary position to a desired trajectory while ensuring the first order of the sliding surface = 0 ($\dot{s} = 0$).

The first-order derivative of the sliding surface (Eq. 3) is

$$\dot{s} = \;\ddot{\tilde{q}} + \lambda\dot{\tilde{q}} = \ddot{q} - \ddot{q}_d + \lambda\dot{\tilde{q}} \tag{5}$$

The plant dynamics (Eq. (1)) and substitution for $\ddot{q}$ yields

$$\dot{s} = \left(\frac{mghsin(q)}{I} + \frac{T}{I}\right) - \ddot{q}_d + \lambda\dot{\tilde{q}} \tag{6}$$

The best estimated ankle torque $\hat{T}$ that makes $\dot{s} = 0$ is thus

$$\hat{T} = I\ddot{q}_d - \lambda I\dot{\tilde{q}} - Mghsin(q) \tag{7}$$

To account for both the imprecision of the system dynamics and chattering behavior, the control law T has to be discontinuous [31], and a sign function of s is introduced as

$$T = \hat{T} - Ksgn(s) \tag{8}$$

where $sgn(s) = +1$ if $s > 0$, $sgn(s) = -1$ if $s < 0$, and K is the control gain.

T can be further decomposed into passive ankle torque (a function of current state) and active ankle torque (a function of desired state and difference of current and desired states)

$$T_{passive} = -\;Mghsin(q) \tag{9}$$

$$T_{active} = I\ddot{q}_d - \lambda I\dot{\tilde{q}} - Ksgn(s) \tag{10}$$

Considering afferent sensory time delay $\Delta t = 200\,ms$ [10], the first-order Taylor expansion of q and $\dot{\tilde{q}}$ becomes [28]

$$q = q - \Delta t\dot{q} \text{ and } \dot{\tilde{q}} = \dot{q} - \;\dot{q}_d = \;\dot{q} - \Delta t\ddot{q} - \dot{q}_d \tag{11}$$

To achieve Lyapunov stability (Eq. (4)), K, the control gain, must be > 0. Here, K was chosen using

$$K = \varepsilon + \beta I\left|\dot{\tilde{q}}\right| \tag{12}$$

where $\varepsilon,\ \beta > 0$.

Considering Eqs. (7), (8), and (1), the following representation holds:

$$I\ddot{q} = I\ddot{q}_d - \lambda I\dot{\tilde{q}} - Ksgn(s) \tag{13}$$

COM angle and angular velocity were initialized with arbitrary values (e.g., $q = -0.2$ and $\dot{q} = 0.1$). Subsequently, experimental COM kinematics, λ I, K, and $sgn(s)$ were input to a differential equation solver (Matlab ODE4). Consistent values of $\varepsilon,\ \beta$ (selected, though an iterative search, to balance tracking error and stability convergence time) were used for all trials ($\varepsilon = 0.8$, $\beta = 0.12$).

Model performance was evaluated in each sway trial using a root-mean-square RMS) tracking error

$$\text{RMS} = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}{n}} \tag{14}$$

where x_i^2 is the squared difference between modeled and experimental values (kinematics and kinetics) and i indexes over the n data points within a trial.

Mean values of the tracking error ratio, and Pearson product-moment correlation coefficients between modeled and experiment values, were obtained within each trial. Mean values of both passive and active predicted ankle torques were also determined, and the associated mean passive/active torque ratio. Phase relationships were obtained between passive ankle torque and COM angle, and between active ankle torque and COM angular acceleration. Summary statistics are reported below across the 96 trials (32 participants, three trials each), as means (standard deviations).