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# A model of lung parenchyma stress relaxation using fractional viscoelasticity

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ABSTRACT

Some pulmonary diseases and injuries are believed to correlate with lung viscoelasticity changes. Hence, a better understanding of lung viscoelastic models could provide new perspectives on the progression of lung pathology and trauma. In the presented study, stress relaxation measurements were performed to quantify relaxation behavior of pig lungs. Results have uncovered certain trends, including an initial steep decay followed by a slow asymptotic relaxation, which would be better described by a power law than exponential decay. The fractional standard linear solid (FSLS) and two integer order viscoelastic models – standard linear solid (SLS) and generalized Maxwell (GM) – were used to fit the stress relaxation curves; the FSLS was found to be a better fit. It is suggested that fractional order viscoelastic models, which have nonlocal, multi-scale attributes and exhibit power law behavior, better capture the lung parenchyma viscoelastic behavior.


## 1. Introduction

Viscoelastic materials are those which behave between elastic solids and viscous fluids. Lung parenchyma, like many biological soft tissues, is highly viscoelastic. A wide range of pulmonary pathologies and injuries such as fibrosis, asthma and emphysema correlate with significant changes, locally or diffusely, in lung viscoelasticity [1–5]. Better understanding of lung viscoelastic properties could provide new perspectives on the relation between lung structure and function, and the progression of pathology and trauma from a biomechanical point of view. By developing an improved understanding of the viscoelastic behavior of biological tissue, more realistic analytical models of biological tissues would be possible. These in turn may enable improved pulmonary diagnostic, therapeutic and educational modalities. Examples of these possibilities include: diagnostic systems based on breath sound analysis; diagnosis utilizing magnetic resonance and ultrasound elastography; therapeutic approaches such as lung cancer particle beam and other radiation therapy; and improved haptic and other simulation technologies to facilitate video assisted thoracoscopic surgery training.

Lung tissue was first noted to be viscoelastic by Bayliss and Robertson in 1939 and by Mount in 1955 [6,7]. Lung stress relaxation and hysteresis were studied by Marshall and Widdicombe [8]. Hildebrandt [9,10] and Bachofen [11] studied lung viscoelasticity in human and in isolated cat lung. Hildebrandt [12] and Suki et al. [13] found that for an isolated cat lung the ratio of pressure and volume followed temporal power law dependence. Suki et al. [13] used fractional viscoelasticity to model the lung tissue mechanics and hypothesized on its molecular basis. Magin [14] provides a review of fractional viscoelasticity modeling and describes its fractal origins making it a rational choice for biological and other materials with a complex multi-scale structure.

Models using mechanical analogies of spring and dashpot components have been used to represent material viscoelastic properties. The parameters of these components can be estimated by least square fitting the measurements of the temporal or spectral response of the material Young's or shear modulus to the predictions of the different models. The standard linear solid (SLS) is the simplest model that predicts stress relaxation and creep with a parallel combination of a Maxwell model (spring and dashpot in series) and a spring [15]. Its temporal response to a step strain is a decaying exponential function. The SLS model shows limitations in its ability to accurately capture dynamic phenomena over multiple time scales and/or with broad spectral content, particularly for biological tissues. One way to overcome such limitations is through the use of more complex models with a larger number of parameters to increase agreement with experimental behavior; but, this comes at the expense of obscuring the physical meaning of the viscoelastic model.

Recently, some models based on the fractional order derivative have been applied to biological tissue viscoelasticity [13,16–21]. The

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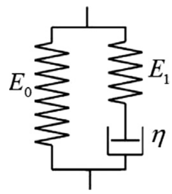


**Fig. 1.** Schematic diagram of SLS model.

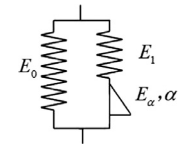


**Fig. 2.** Schematic diagram of FSLS model.

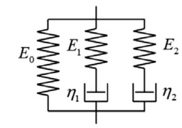


**Fig. 3.** Schematic diagram of generalized Maxwell model.

fractional order derivative leads to a component called a spring-pot of order $\alpha$ which behaves between pure elastic and viscous materials. Both temporal relaxation and frequency response of a spring-pot follow power law functions that seem to be naturally adapted to fit soft tissue viscoelasticity. Craiem et al. [22] performed uniaxial elongations on human arteries and found that a fractional SLS model predicted arterial stress relaxation better than the conventional SLS model. Fractional order viscoelastic models also have shown the potential to yield new disease and treatment specific parameters that more effectively predict underlying changes in tissue associated with developing pathology, such as liver cirrhosis and breast cancer. For example, a relatively simple power law relationship was fit to the complex shear modulus of human breast tissue and tumors measured by magnetic resonance elastography [21]. The results, when plotted as the fractional power exponent versus the fractional order attenuation, separated benign from malignant tumors with an increased specificity and sensitivity compared to other models.

The objective of the current study is to measure the stress relaxation on freshly excised and inflated pig lungs and fit the measurement with typical models: SLS, GM and FSLS. The degree of fitting, and advantages and disadvantages of each model will be evaluated. The stress-strain relationship and relaxation function due to a unit step strain are covered in Section 2. The experimental procedures including: fresh and inflated pig lung preparation, the stress relaxation test, and mechanical indentation test are described in Section 3. Section 4 displays the experimental and fitting results followed by a discussion in Section 5.

## 2. Theory

According to Fung [15], the stress-strain relationship for the standard linear solid (SLS) model (shown in Fig. 1) is

$$\frac{d\varepsilon(t)}{dt} = \frac{E_1}{\eta(E_0+E_1)}\left[\frac{\eta}{E_1}\frac{d\sigma(t)}{dt} + \sigma(t) - E_0\varepsilon(t)\right] \tag{1}$$

where $\sigma$ is the stress, $\varepsilon$ is the strain, $t$ is the time, $E_0$ and $E_1$ are the spring stiffness in the SLS model and $\eta$ is the damping coefficient of the dashpot. Subject to a strain of a unit step function 1(t) and according to Fung [15], the relaxation function, which is the stress resulting from that strain, is

$$G(t) = (E_0 + E_1 e^{-t/\tau})1(t) \tag{2}$$

where $\tau = \eta/E_1$ and is called the relaxation time for constant strain.

According to Craiem et al. [22], the fractional order derivative $\alpha$ of a function f(t) can be expressed by the following integral representation

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt} = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{f(\tau)}{(t-\tau)^\alpha}d\tau \tag{3}$$

where $D^\alpha$ denotes the $\alpha$th order derivative of *f(t)* with respect to time, and $\Gamma$ is the gamma function. By using fractional order derivatives, we can create a component called a spring-pot of order $\alpha$ that behaves between pure elastic and viscous materials. The stress strain relationship of a spring-pot is

$$\sigma(t) = E_\alpha D^\alpha \varepsilon(t) \tag{4}$$

where $E_\alpha$ is the viscoelastic coefficient. Substituting the dashpot in the SLS model with a spring-pot leads to the fractional SLS (FSLS) model shown in Fig. 2 and according to Craiem et al. [22] the stress strain relationship is

$$D^\alpha\varepsilon(t) = \frac{E_1}{E_\alpha(E_0+E_1)}\left[\frac{E_\alpha}{E_1}D^\alpha\sigma(t) + \sigma(t) - E_0\varepsilon(t)\right] \tag{5}$$

Subject to a strain of a unit step function 1(t) and according to Craiem et al. [22], the relaxation function of the FSLS model is

$$G(t) = (E_0 + E_\alpha F_\alpha[-(t/\tau_\sigma)^\alpha])1(t) \tag{6}$$

where $\tau_\sigma = (E_\alpha/E_1)^{1/\alpha}$ and $F_\alpha$ is the Mittag–Leffler function defined as

$$F_\alpha(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)} \tag{7}$$

If the spring $E_1$ is removed in the FSLS model this leads to the fractional Voigt (FV) model. According to Magin [14], its relaxation function due to unit step function shows the power law relaxation and it is

$$G(t) = \left(E_0 + E_\alpha\frac{t^{-\alpha}}{\Gamma(1-\alpha)}\right)1(t) \tag{8}$$

It is observed that there is a singularity at t = 0. So the FV model was not used for stress relaxation fitting in the current study due to this singularity even though it is widely used in fitting the dynamic modulus in the frequency domain.

The generalized Maxwell (GM) model with two Maxwell elements assembled in parallel is shown in Fig. 3. Its relaxation function is

$$G(t) = (E_0 + E_1 e^{-t/\tau_1} + E_2 e^{-t/\tau_2})1(t) \tag{9}$$

where $\tau_1 = \eta_1/E_1$, $\tau_2 = \eta_2/E_2$. $E_0$, $E_1$ and $E_2$ are the spring stiffness. $\eta_1$ and $\eta_2$ are the damping coefficients of the dashpots. $\tau_1$ and $\tau_2$ are the different relaxation times for constant strain.

In the current study, the ability of different models to describe stress relaxation of the pig lung is assessed. Model selection should be done with care. The selection criterion is that the model should accurately capture the lung stress relaxation with the fewest number of parameters, which can render an easier physical interpretation. It is reported that for most soft biological tissues subject to step function strain, the relaxation stress is finite and asymptotically reaches a steady state non-zero value [13,23,24]; so models with stress singularity at t = 0 or those with zero stress for a large times should be excluded. Considering two-parameter models, it can be seen that the Voigt model has stress singularity at t = 0 and the Maxwell model has an exponentially decaying stress to a zero value. For a spring-pot model, the stress has a singularity at t = 0 and asymptotically approaches zero. Among the three-parameter integer order models shown in Fig. 4, the first two are called Zener models and the last two are called anti-Zener models by Mainardi and Spada [25]. For Zener