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Technical Note

Stress free configuration of the human eye

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ABSTRACT

Numerical simulations of eye globes often rely on topographies that have been measured in vivo using devices such as the Pentacam or OCT. The topographies, which represent the form of the already *stressed* eye under the existing intraocular pressure, introduce approximations in the analysis. The accuracy of the simulations could be improved if either the stress state of the eye under the effect of intraocular pressure is determined, or the stress-free form of the eye estimated prior to conducting the analysis. This study reviews earlier attempts to address this problem and assesses the performance of an iterative technique proposed by Pandolfi and Holzapfel [1], which is both simple to implement and promises high accuracy in estimating the eye's stress-free form. A parametric study has been conducted and demonstrated reliance of the error level on the level of flexibility of the eye model, especially in the cornea region. However, in all cases considered 3–4 analysis iterations were sufficient to produce a stress-free form with average errors in node location $<10^{-6}$ mm and a maximal error $<10^{-4}$ mm. This error level, which is similar to what has been achieved with other methods and orders of magnitude lower than the accuracy of current clinical topography systems, justifies the use of the technique as a pre-processing step in ocular numerical simulations.

1. Introduction

The human eye is a complex optical system whose three-dimensional shape determines the clarity of vision. Light rays entering the eye are refracted four times before they reach the retina; twice at the two surfaces of the cornea, which accounts for about two-thirds of the refractive power, and twice at the surfaces of the crystalline lens. Small geometric changes such as imperfections of the cornea or elongation of the sclera may lead to a blurry and/or distorted vision. Nowadays, several surgical interventions induce changes in corneal [2] and scleral [3] shapes in order to improve vision. In conjunction with the advancement of surgical techniques, numerical models have been proposed as tools to better understand the effect of surgeries on the eye [4–8]. To aid these efforts, experimental investigations have been conducted to characterize the mechanical properties of the cornea [9] and sclera [10], and quantify their non-linear, anisotropic and visco-elastic behavior [11,12].

The 3D ocular topography can also be measured in vivo with several devices such as the Pentacam, Gallilei or OCT. However, since the eye supports intraocular pressure (IOP), the shape measured experimentally corresponds to a deformed configuration with the tissue under primarily membrane tension. This *deformed* configuration is unsuitable for direct implementation in patient-specific models as the application of IOP on the models would result in a larger shape than that obtained experimentally. If on the other hand the IOP is not applied, its associated stresses will not be generated in the model, which also affects the model's representation of in vivo conditions. For these reasons, it is necessary for accurate numerical modeling to determine the relaxed (stress-free) configuration of the tissue, or the stable eye geometry that would be adopted had the IOP been removed.

Two main techniques have been proposed to solve this problem. Some studies adopted an inverse elastostatic approach based on the reverse application of IOP to explicitly estimate the stress-free configuration [13–15]. This was followed by the application of IOP on the estimated stress-free form to determine the resulting stresses in the deformed configuration through the solution of a forward problem. One of the limitations of this approach is that the thin-walled flexible ocular structure may exhibit bifurcation during the backward calculation of the zero pressure geometry [16]. To solve this problem, Gee et al. proposed a modified updated Lagrangian formulation to estimate the stress in the tissue using a forward calculation [17], and a similar technique was used by Lanchares et al. to

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pre-stress a cornea model to simulate arcuate keratotomy surgical intervention [18]. This approach, which attempts to avoid the need to determine the stress-free configuration, was later refined by Grytz and Downs to improve its accuracy [16]. The main difference between the two methods is that in inverse elastostatic approach, a complete simulation step is required to update the deformation gradient tensor while the framework proposed by Grytz and Downs continuously updates the deformation tensor during the application of the load on the eye model.

Although these methods have been shown to provide accurate results, their implementations remain complicated and require a thorough understanding of continuum mechanics and finite element analysis. An alternative approach has been proposed by Pandolfi and Holzapfel [1], where general-purpose finite element software packages are combined with simple mathematical calculations to obtain stress-free configurations. The objective of this study is to assess this approach and determine if it provides a level of accuracy sufficient for ophthalmic applications.

2. Methods

The method used in this study is based on the simulation of ocular deformation with finite element techniques. An iterative approach is used to gradually move the nodes of the finite element mesh to their stress-free configuration.

2.1. Calculation of the stress-free configuration

An iterative approach based on work of Pandolfi and Holzapfel [1] is used to obtain the stress-free configuration. An initial numerical model is built based on a measured (stressed) shape of the tissue X_0. In a first step, the displacements u_1 induced by the intraocular pressure (IOP) are calculated using non-linear finite element analysis. These displacements vectors are then subtracted from the node coordinates of the stressed shape to determine the first estimate of the stress-free form X_1, Fig. 1.

$$u_1 = X_0 - x_0 \tag{1}$$

$$X_1 = X_0 - u_1 \tag{2}$$

In a second step, the model with the initial stress-free form X_1 is re-inflated with IOP and the error vector u_2 between the resulting (deformed) nodal positions x_1 and the target nodal positions X_0 (corresponding to the measured stressed configuration) is calculated at all nodes. The error vector is then subtracted from the node coordinates of the current assumed stress-free form X_1 to define the initial configuration X_2 for the following analysis iteration.

$$u_2 = X_0 - x_1 \tag{3}$$

$$X_2 = X_1 - u_2 \tag{4}$$

This process is repeated while monitoring the values and distribution of nodal errors relative to the target (measured) configuration.

$$u_k = X_0 - x_{k-1} \tag{5}$$

$$X_k = X_{k-1} - u_k \tag{6}$$

The absolute nodal distance and the root mean square distance (L^2-norm) are used to quantify the maximum and average error and to monitor the convergence of the iterative process toward the stress-free form. The error estimate of the kth increment in then given by:

$$e_k = ||X_0 - x_k|| \tag{7}$$

2.2. Finite element model of the eye

In order to evaluate the accuracy of the proposed approach, a parametric model of the human eye was developed. The model incorporated a number of features to closely represent in vivo conditions including the cornea's and sclera's non-uniform thickness, asphericity of both the cornea's anterior and posterior surfaces, and weak inter-lamellar adhesion within corneal stroma. The finite element (FE) software ABAQUS 6.10 (Dassault Systèmes Simulia Corp., Rhode Island, USA) was used in the analyses. The model used six-noded solid elements (C3D6H) to enable a good representation of the variable thickness. The elements were arranged in either 1, 2 or 4 layers and 120 rings, 20 of which were in the cornea, to provide smooth distributions of strains and displacements, see Fig. 2. The total elements numbers were 6962, 13,924 or 27,848 for the case with 1, 2 and 4 layers, respectively. Weak stromal inter-lamellar adhesion was assumed only in the cornea and in models with 2 or more layers, and was set to the level observed experimentally [19]. This feature could not be considered in the model with only one element layer.

The models incorporated a material definition that combined both the hyperelasticity and hysteresis of the tissue as has been detailed in Elsheikh et al. [9,10,20]. With these two characteristics, the material experienced a gradually increasing stiffness under load, and should the material be unloaded, it would follow a different stress–strain path [20]. With both behavior patterns described in the material definition, the simulation would be able to select which stress–strain path each element was to follow based on its strain history. Moreover, varying the age in the model meant changing the constitutive model parameters of the cornea and sclera based on the relationships provided by Elsheikh et al. [9,10]. The constitutive model used to represent corneal and scleral mechanical behavior for all ages was the third order Ogden hyperelastic model.

Further, the models were provided with support conditions designed to avoid interference with the behavior under IOP. The models were prevented from displacement in the anterior–posterior direction (z-direction) along all equator nodes and displacement in the superior–inferior and temporal–nasal directions at the corneal apex and posterior pole. Only a quarter of the eye was used in this work, for that the symmetry boundary conditions were applied at the two ends (Fig. 2).

2.3. Parametric study

The accuracy of the method has been evaluated in a parametric study that considered eye numerical models with variations in corneal anterior radius, R, shape factor, p, sclera radius, R_s, intraocular pressure, IOP, and age. The parameter variations shown in Table 1 have been chosen to cover, and in some cases go beyond, the variability present in the general population. A reference benchmark model, with the parameter baseline values in gray cells in Table 1, was considered. Within each part of the study, only one parameter was allowed to vary while other parameters maintained their baseline values as in the benchmark model.

All models considered a central corneal thickness, CCT, of 545 μm – the average reported value in a number of earlier studies [21] – and a peripheral corneal thickness, PCT, that was larger than CCT by 150 μm in line with Gullstrand's No 1 schematic eye [22]. While the cornea was considered to be an ellipsoid with variable central radius and shape factor, the sclera assumed a spherical outer surface and a thickness that changed from PCT at the limbus, reducing gradually to 0.8 PCT at the equator before increasing to 1.2 PCT at the posterior pole, based on the results of an earlier study [10].