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Evaluation of the optimal lengths and number of multiple windows for spectrogram estimation of SSVEP

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ABSTRACT

The purpose of this paper is to present the optimal number of windows and window lengths using multiple window spectrogram for estimation of non-stationary processes with shorter or longer duration. Such processes could start in the EEG as a result of a stimuli, e.g., steady-state visual evoked potentials (SSVEP). In many applications, the Welch method is used with standard set-ups for window lengths and number of averaged spectra/spectrograms. This paper optimizes the window lengths and number of windows of the Welch method and other more recent, socalled, multiple window or multitaper methods and compares the mean squared errors of these methods. Approximative formulas for the choice of optimal number of windows and window lengths are also given. Examples of spectrogram estimation of SSVEP are shown.

1. Introduction

Estimation and detection of frequency changes of shorter or longer duration in the EEG, connected to stimuli, are often of great interest. For a repetitive periodic visual stimulus a steady-state visual evoked potential (SSVEP) arises. The estimation and detection of SSVEP is of interest in many applications, e.g., in working memory tasks [1–3], brain–computer interfaces [4–6], and others. Different approaches to estimate and model the SSVEP have been taken over the years, e.g., [5,7–9]. However, many of these models are too deterministic to be reliable and recent research shows that the visual evoked potentials are a combination of additive events and phase changes of the EEG, [9,10]. Therefore a simple stochastic model is adopted in this paper, where a few parameters determine the behavior of the event.

Usually a spectrum or spectrogram approach based on a single window is used for estimation of evoked potentials. Sometimes it is also argued that no window at all should be used [11]. This is usually based on that SSVEP has a deterministic behavior and the periodogram (without window) has also been shown to be optimal for detection of a sinusoid frequency disturbed by noise, [12]. However, for stochastic processes the periodogram has a large variance and if we assume the SSVEP to be a stochastic process the periodogram is not necessarily the optimal solution.

The large variance is reduced using averaging of uncorrelated spectra [13], where the uncorrelated spectra come from different non-overlapping or partly overlapping data sequences. The method is mostly called the Welch method or WOSA (weighted overlapped segment averaging) and a common data overlap is 50% which has been shown to be appropriate from resolution and variance aspects. Parametric spectra, based on AR and ARMA models, have been used frequently but are well known to be unreliable with spurious peaks and other estimation errors.

More recently, the use of socalled *multiple windows* has been proposed. The concept of multiple windows was introduced by Thomson [14], and the method has been shown to give a better result than the WOSA method in terms of leakage, resolution and variance [15]. For multiple windows, the properties that give uncorrelated spectra come from the windows and not from data. Multiple windows make use of all data samples for all windows and are totally overlapping and thereby they use more of the information in data than, e.g., the WOSA method. The optimal bias and variance reduction from multiple windows can be achieved utilizing appropriate windows for the properties of data. With certain constraints on data, e.g., frequency local white spectrum for the Thomson method [14], the window shapes are designed to give uncorrelated

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subspectra. For a varying spectrum, e.g., a large dynamic spectrum with peaks and notches, however, the performance of the Thomson method degrades due to cross-correlation between spectra [16]. The minimum bias multiple windows and the sinusoid windows in [17] and the peak matched multiple windows (PM MW) [18,19], have better properties for frequency-varying spectra.

Multiple window spectrograms have been proposed and evaluated for many different application areas and also for the estimation of EEG, e.g., Hermite functions and the Thomson multiple windows in [20], where it was shown that the time–frequency localization is higher for the Hermite functions compared to the Thomson windows. The Thomson multiple windows are also applied in [21] for characterization of brain oscillations where it was noted that window lengths and number of windows were important parameters for the characterization. The PM MW have also been evaluated and compared to WOSA and the Thomson windows for estimation of EEG-spectra in [22]. Events of different kinds that arise in the EEG, often connected to some stimulus, call for robust estimation methods that are adapted to the events. The events have typically a non-stationary behavior and the multiple windows need to be optimal for such events.

Time–frequency analysis, for *time and frequency* varying spectra, include different approaches, e.g., the short time Fourier transform (STFT) or spectrogram, the Wigner-Ville distribution (WVD) as well as the wavelet transform. Recently, multiple windows are introduced for non-stationary stochastic processes. The STFT is appropriate in many cases where resolution is important and can be seen as a special case of the multiple window techniques with just one window. The WVD is well known to have drawbacks, as the cross-terms usually destroy the information for stochastic processes. For the estimation of the time- and frequency properties of the SSVEP an approach using, e.g., wavelets is not that appropriate, contrary to the estimation of shorter event-related signals ≈ 1 s. Other methods are proposed, e.g., the chirplet transform in [23] and multiple windows that are optimal for a class of locally stationary processes (LSP MW), in [24]. This class of processes spans a wide range different time- and frequency bandwidths.

The LSP MW are very appropriate for robust estimation of non-stationary processes as just one parameter determines the whole set of multiple windows and weighting factors and the set of multiple windows can be well approximated as a set of Hermite functions which is an advantage from computational aspects. Above this, the same set of Hermite functions can be used for all frequency bandwidths, meaning that a number of windowed spectrograms can be computed and stored. For the final estimate, the windowed spectrograms are weighted together according to an optimal parameter set. Thereby, if there are inaccuracies in the modeling of data, a wider set of optimal estimators in the class of locally stationary processes are easily evaluated. This flexibility also makes the estimator to actually approximately vary from a STFT-estimator (one spectrogram) to a WVD-estimator (averaging of a large number with alternating sign of the weights). The LSP MW are applied for estimation of events in the EEG in [25].

In this paper, the mean squared error optimal window lengths and number of windows are computed and compared for the WOSA, the PM MW, the Hermite functions and the LSP MW, in multiple window spectrograms of an event, e.g., a SSVEP. In Section 2, some simplified expressions for the mean squared error of multiple window spectrogram estimators are suggested. Based on these expressions, closed form formulas for the approximative window lengths and number of windows are given. In Section 3, the multiple window spectrogram and the different methods are defined and Section 4 presents the theoretical formulas which are used in the evaluation. In Section 5 some real data examples are given and some general conclusions for the use of multiple windows in spectrogram estimation of SSVEP are presented in Section 6.

2. Estimation of approximate window lengths and number of windows for multiple window spectrograms

To accurately estimate the power distribution in frequency of a real-valued sequence of a stationary process $\{x(m), m = 0, \ldots, N-1\}$ with spectrum $S(f)$, the mean squared error (MSE), including variance and squared bias measures, could be used, i.e.,

$$E[(\hat{S}(f) - S(f))^2] = V[\hat{S}(f)] + B^2(f), \qquad (1)$$

where the variance is defined as

$$V[\hat{S}(f)] = E[(\hat{S}(f))^2] - E^2[\hat{S}(f)],$$

and the squared bias as

$$B^2(f) = (E[\hat{S}(f)] - S(f))^2.$$

For the *windowed periodogram*,

$$\hat{S}(f) = \frac{1}{N}|\sum_{m=0}^{N-1} x(m)h(m)e^{-j2\pi fm}|^2, \qquad (2)$$

where $\{h(m), m = 0, \ldots, N-1\}$, is a window function, the variance is usually approximated as

$$V[\hat{S}(f)] \approx S^2(f), \quad 0 < f < 0.5, \qquad (3)$$

and the expected value is

$$E[\hat{S}(f)] = \int_{-1/2}^{1/2} S(u)K_N(f-u)du, \qquad (4)$$

where for the rectangle window, $\{h(m) = 1, m = 0, \ldots, N-1\}$,

$$K_N(f) = \frac{\sin^2(N\pi f)}{N\sin^2(\pi f)},$$

is known in literature as *Fejér's kernel*. The half-value *frequency bandwidth*, i.e., where $K_N(B_w/2) = 1/2$ if $K_N(0) = 1$, is approximately $B_w = 1/N$, which is said to be the spectral resolution limit. The half-value *time bandwidth* of the rectangle window of length N is $T_w = N$ but for the Hanning window we instead find $T_w = N/2$ and $B_w = 2/N$. Conclusively, a time–frequency bandwidth is found as $T_wB_w = 1$ [26, p. 42].

For a multiple window estimator based on, e.g., the Thomson multiple windows, $\{h_i(m), m = 0, \ldots, N-1\}, \quad i = 1 \ldots I$,

$$\hat{S}(f) = \frac{1}{I}\sum_{i=1}^{I}|\sum_{m=0}^{N-1} x(m)h_i(m)e^{-j2\pi fm}|^2, \qquad (5)$$

the spectral resolution is approximately equal to $B_w = I/N$, [14], and as these windows vary considerably in time, we use the time-resolution of the rectangle window above, $T_w = N$, giving $T_wB_w = I$. The variance is, for a smooth spectrum, approximately,

$$V[\hat{S}(f)] \approx \frac{1}{I}S^2(f), \quad 0 < f < 0.5. \qquad (6)$$

This formula is approximately valid also for, e.g., the WOSA with 50% overlap between the windows.

In this paper, a simple AR(2)-model is proposed for the studied process. The damping factor ρ_0 is varied to simulate different types of processes, see Fig. 1, where two different realizations are shown in each plot for different ρ_0 and sequence length T in seconds. The frequency local shape of the AR(2)-spectrum can be approximated with a similar AR(1)-model with spectrum,

$$S(f) = \frac{(1-\rho_0)^2}{1+\rho_0^2 - 2\rho_0\cos(2\pi f)},$$