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Poroelastic behaviour of cortical bone under harmonic axial loading: A finite element study at the osteonal scale

Vu-Hieu Nguyen, Thibault Lemaire, Salah Naili*

Université Paris-Est, Laboratoire Modélisation et Simulation Multi Échelle, UMR 8208 CNRS, Faculté des Sciences et Technogie, 61, avenue du Général de Gaulle, 94010 Créteil cedex, France

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ABSTRACT

Bone fluid flow and its induced effects on the bone cells are important players in triggering and signalling bone formation and bone remodelling. This study aims to numerically investigate the behaviour of interstitial fluid flows in cortical bone under axial cyclic harmonic loads that mimics *in vivo* bone behaviour during daily activities like walking. Here, bone tissue is modelled as a fluid-saturated anisotropic poroelastic medium which consists of a periodic group of osteons. By using a frequency-domain finite element analysis, the fluid velocity field is quantified for various loading conditions and bone matrix parameters.

1. Introduction

Cortical tissue is the dense part of bone. As a living entity, this material is able to maintain and adapt its structure to external physical *stimuli* [11]. The seat of bone remodelling mechanisms corresponds to cylindrical structural elements called osteons. Each osteon is surrounded by a thin layer (cement line) and is centered on Haversian canal which runs primarily in the bone longitudinal axis. The Haversian canals contain the vasculature, the nerves and interstitial fluid. There are also Volkmann canals which are similar to Haversian canals except that they run along the transverse direction of the bone. At a smaller scale, other extravascular pores exist in the solid matrix of the bone forming the lacuno-canalicular system. This porous network irrigates the mechano-sensitive osteocytes which are believed to play an important role in bone adaptation as stated in recent experimental studies [7,18,16,25].

The study of the macroscopic mechanical behaviour of bone is useful in order to describe the hydraulic response in the vicinity of cells which is a subject of great interest since it could help to better understand bone remodelling [21,17,9,8]. By assuming an impermeable cement line, our group proposed several studies of the poroelastic response of an isolated osteon subjected to mechanical loading [14,12,15,13]. However in reality, micropores of osteonal tissue may cross cement lines [6] and thus the cement line may not be quite impermeable. Although the question about values of the cement line's permeability is still opened, such a pervious property of the cement line is expected to modify the hydro-mechanical behaviour of cortical tissues.

Hence, the main purpose of this study is to extend our previous studies to take into account the possibly existing flows through the cement coating surfaces. We are also interested in taking into account the interaction between the considered osteon and the environment around it. For this purpose, this study proposes to describe the cortical tissues by several secondary osteons, each of which is coated by a cement line, embedded in a matrix made of old osteonal tissues. An idealized model is carried out to mimic the hydro-mechanical behaviour of this system and analyze the influence of loading parameters, cement line permeability and geometry characteristics. Using the model, it is proposed to test the hypothesis that the interstitial fluid flow is independent of fluid flow but dependent on rate of strain. Further it is proposed to test the hypothesis that cement line permeability does not contribute to interstitial fluid field.

This paper is organized as follows. After this introduction on the rationale of the behaviour of interstitial fluid flows in cortical bone under axial cyclic harmonic loads, Section 2 introduces the model considering the bone matrix as a periodic array of osteons. Based on Biot's poroelastic theory applied to three-dimensional anisotropic media, the governing equations are presented. Next, we propose an equivalent two-dimensional system by assuming a homogeneous deformation in the longitudinal direction and focusing on a domain far from transverse vasculature. In Section 3, we develop the problem in the frequency domain and derive the corresponding weak

* Corresponding author. Tel.: +33 1 45 17 14 45; fax: +33 1 45 17 14 33.
E-mail address: salah.naili@univ-paris-est.fr (S. Naili).

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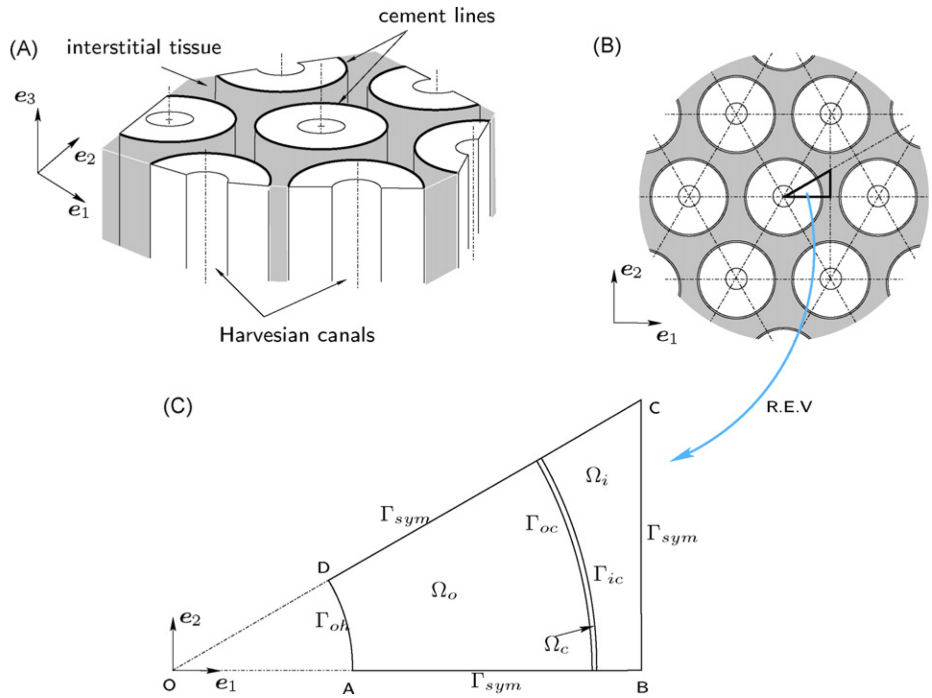

Fig. 1. Cortical tissue: (A) three-dimensional representation; (B) two-dimensional projection; (C) representative elementary volume.

formulations. Finally, Section 4 provides some numerical results of the fluid velocity, considering different geometrical and textural properties of cortical tissue under various loading conditions. The interest of these results for bone remodelling, mechanotransduction and cell stimulation is also discussed.

2. Description of the configuration and formulation of the problem

2.1. Geometrical configuration

In the osteonal bone matrix, Haversian canals run longitudinally through the bone cortex and are transversely inter-connected by Volkmann canals. Each osteon is developed concentrically around one Haversian canal and presents a cylinder-like form. For simplification purposes, the osteonal zone that is considered here is assumed to be far enough from transverse Volkmann canals, so that the influence of these canals can be neglected. Without this assumption, a large complex 3D model would have to be done.

We consider a representative matrix of osteons containing Haversian canals (see Fig. 1(A)). Let $\mathbf{R}(O;\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ be the Cartesian frame of reference where O is the origin of the space equipped with an orthonormal basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$. The osteons, which all run in the vertical direction $\mathbf{e}_3$, are modelled as thick-walled hollow cylinders. They are assumed to be identical and parallel. Moreover, they are arranged periodically in the horizontal plane $(\mathbf{e}_1,\mathbf{e}_2)$ (see Fig. 1(B)). Each osteon is coated by a thin layer called the cement line. The tissue found outside of the cement lines, *i.e.* the tissue that fills the space between the osteons, is the osteonal matrix formed by the remnants of old osteons.

The position $\mathbf{x}$ of the particle of the medium is specified through the coordinates (x_1,x_2,x_3) with respect to $\mathbf{R}$. The time is denoted by t. Moreover, the Einstein summation convention, which stipulates that repeated indices are implicitly summed over, is used.

In what follows, superscripts referring to different material components of the cortical medium are introduced: Haversian canal (h), osteon (o), cement line (c) and interstitial tissues (i).

2.2. Three-dimensional governing poroelastic equations

The bone tissue materials (osteons, cement lines and interstitial matrix) are considered as saturated anisotropic poroelastic media. Neglecting body forces, the governing poroelastic equations for anisotropic material in the low frequency range are given by [1,3]:

$$\rho\ddot{\mathbf{u}} - \operatorname{div}\boldsymbol{\sigma} = \mathbf{0}, \tag{1}$$

$$\frac{1}{M}\dot{p} - \operatorname{div}(\mathbf{k}\operatorname{grad}p) + \boldsymbol{\alpha}:\dot{\boldsymbol{\varepsilon}} = 0, \tag{2}$$

where $\rho = \phi\rho_f + (1-\phi)\rho_s$ is the mixture density which is defined from the porosity ϕ and the densities ρ_f and ρ_s of the fluid and solid phases, respectively; $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ are the displacement vector and the strain tensor of the solid skeleton, respectively; $\boldsymbol{\sigma}$ is the total stress tensor; p is the fluid pressure in saturated pores; $\mathbf{k}$ is the anisotropic permeability tensor; $\boldsymbol{\alpha}$ is the Biot tensor and M is the Biot *modulus*. The operators div and grad are respectively the divergence and gradient. Differentiation with respect to time t is denoted by superposed dot and the symbol ':' between tensors of any order denotes their contraction.

Note that the permeability $\mathbf{k}$ is the textural parameter allowing to quantify the ability of a porous material to transmit fluids through the Darcy law:

$$\mathbf{v} = -\mathbf{k}\operatorname{grad}p, \tag{3}$$

where $\mathbf{v}$ is the filtration velocity vector defined by $\mathbf{v} = \phi(\dot{\mathbf{u}}^f - \dot{\mathbf{u}})$ where $\dot{\mathbf{u}}^f$ is the velocity of the interstitial fluid. The tensor $\mathbf{k}$ may be evaluated by $\mathbf{k} = \boldsymbol{\kappa}/\eta$ where $\boldsymbol{\kappa}$ is the intrinsic permeability and η the pore fluid dynamic viscosity.

The stress tensor $\boldsymbol{\sigma}$ is linearly related to the skeleton strain $\boldsymbol{\varepsilon}$ of the porous solid and to the fluid pressure p. Therefore, the constitutive relationship is given by:

$$\boldsymbol{\sigma} = \mathbb{C}:\boldsymbol{\varepsilon} - \boldsymbol{\alpha}p, \tag{4}$$

where $\mathbb{C}$ is the stiffness tensor of the drained material.

For an orthotropic material, $\mathbb{C}$ is defined by nine independent parameters and $\boldsymbol{\alpha}$ is a diagonal tensor defined by three independent