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A mathematical model for creep, relaxation and strain stiffening in parallel-fibered collagenous tissues

Ratchada Sopakayang[a], Raffaella De Vita[b,*]

[a] Department of Mechanical Engineering, Faculty of Engineering, Ubon Ratchathani University, Warinchumrap, Ubon Ratchathani 34190, Thailand
[b] Mechanics of Soft Biological Systems Laboratory, 202 Norris Hall, Department of Engineering Science and Mechanics, Virginia Tech, Blacksburg, VA 24061, USA

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ABSTRACT

A simple model is presented for the description of relaxation, creep, and strain stiffening phenomena that are observed in parallel-fibered collagenous tissues such as ligaments and tendons. In the model formulation, the tissues are assumed to be composed of collagen fibers aligned along their physiological loading direction. The collagen fibers are gradually recruited under strain and are arranged in parallel with a Maxwell element which accounts for the viscoelasticity of the proteoglycan-rich matrix. Once straight, the collagen fibers are assumed to behave as linear elastic springs. Experimental data published by Hingorani et al. [1] are used to estimate the five model parameters by fitting relaxation and strain stiffening data and the predictions are evaluated by using creep data. The influence of each parameter on describing relaxation, creep, and strain stiffening is presented. The modeling results demonstrate that, by considering the fibers' recruitment and assuming that the matrix is linear viscoelastic, a conceptually simple model can describe relaxation, creep, and strain stiffening phenomena in ligaments and tendons.

1. Introduction

Collagenous tissues such as ligaments and tendons are characterized by long-term viscoelastic properties. They exhibit a slow continuous increase in strain over time, or creep, when subjected to a constant stress and a slow continuous decrease in stress over time, or stress relaxation, when subjected to a constant strain. The micro-structural origin of the long-term viscoelasticity of these tissues is still unknown and subject of debate among experts in biomechanics. Synchrotron X-ray scattering studies coupled with mechanical testing have indicated that the collagen fibers, which constitute the main load bearing components of the tissues, may be intrinsically viscoelastic [2] and that the interface between the collagen fibers and the surrounding proteoglycan-rich matrix may also determine the viscoelasticity of the tissues [3]. In many studies, however, the viscoelasticity of the tissues has been attributed to the proteoglycan-rich matrix that surrounds the collagen fibers [4–7].

The difference in the experimental findings suggests the need for more studies aimed at understanding the mechanisms that control the long-term viscoelasticity in ligaments and tendons. As suggested early by Fung [8] and experimentally observed by Thornton et al. [9] and Gupta et al. [7], different structural components of the tissues and their organization are responsible for different viscoelastic phenomena: the recruitment of collagen fibers governs creep [9] while sliding between collagen fibrils/fibers due to the presence of the proteoglycan-rich matrix is predominant in relaxation [7]. Together with histo-mechanical experiments, mathematical models that are formulated by accounting for the micro-structure of collagenous tissues can help in elucidating the relative role of different components of these tissues in determining their long-term viscoelasticity.

The most successful viscoelastic models for creep and relaxation in collagenous tissues are the quasi-linear viscoelastic (QLV) models introduced by Fung [8]. Despite their enormous success, the QLV models have been shown to have limitations since they cannot account for creep rate and relaxation rate dependency as exhibited by ligaments at high stress and strain levels, respectively [10] and, most importantly, cannot interrelate creep and relaxation [11,12]. Nonlinear viscoelastic theories, such as Schapery's theory and the modified superposition method, have been proposed to overcome some of the limitations of the QLV models [13]. Both the QLV models and the newly proposed models are, however, phenomenological models with parameters that lack physical meaning and do not relate to the micro-structural changes that are associated with creep and relaxation.

The long-term viscoelasticity of ligaments and tendons has been described by several structurally based constitutive models [9,14–22]. However, only in the linear viscoelastic model proposed by Thornton et al. [9] creep was predicted from relaxation and this

* Corresponding author. Tel.: +1 540 231 5905.
E-mail addresses: ratchada@vt.edu (R. Sopakayang), devita@vt.edu (R. De Vita).

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was accomplished by accounting for the recruitment of collagen fibers. By performing histological studies, the authors observed that the collagen fibers were gradually recruited during creep due to the increase in strain over time. Moreover, they found that during relaxation only a discrete group of fibers was recruited at the fixed constant strain. Their model was, however, formulated by assuming a specific geometry for the ligaments and arrangement of the wavy collagen fibers.

A simple constitutive framework for modeling relaxation and creep including the strain stiffening in parallel-fibered collagenous tissues is presented. The collagen fibers are assumed to be oriented along the physiological direction of loading. They gradually lose their waviness and become straight under strain at which point they behave as linear elastic springs. The collagen fibers are arranged in parallel with the surrounding matrix that exhibits a Maxwell-type viscoelastic behavior. The model parameters that define the relaxation and strain stiffening phenomena are estimated by using published experimental data by Hingorani et al. [1] on rabbit medial collateral ligaments and are then used to predict creep. The influence of each parameter on describing the long-term viscoelastic properties of collagenous tissues is also analyzed.

2. Model formulation

In this study, the overall viscoelastic behavior of ligaments and tendons is assumed to be determined by their major components: the collagen fibers and the intervening proteoglycan-rich matrix. The collagen fibers are assumed to be aligned along the direction of loading. They are wavy when unstrained and become gradually straight as the overall tissue's strain increases. After becoming straight, the collagen fibers behave as linear elastic springs with equal elastic modulus. The proteoglycan-rich matrix is assumed to behave as a Maxwell-type viscoelastic material, which is described by a linear elastic spring and linear viscous dashpot arranged in series. A schematic of the proposed model, which is described in detail hereafter, is shown in Fig. 1.

2.1. Modeling framework

Ligaments and tendons are modeled as parallel arrangements of linear elastic collagen fibers, each having different waviness, and a linear viscoelastic proteoglycan-rich matrix. Then, the total stress of the tissue, $\sigma(t)$, where t denotes the time, is given by

$$\sigma(t) = \sigma_f(t) + \sigma_m(t), \tag{1}$$

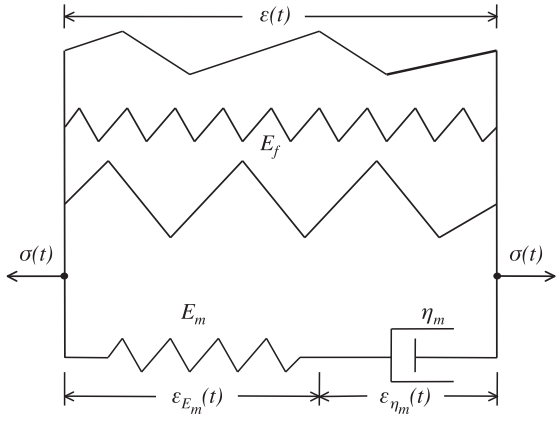

Fig. 1. Schematic of the viscoelastic model.

where $\sigma_f(t)$ is the stress of the collagen fibers and $\sigma_m(t)$ is the stress of the matrix. Moreover, the strain of the tissue, $\varepsilon(t)$, is

$$\varepsilon(t) = \varepsilon_f(t) = \varepsilon_m(t), \tag{2}$$

where $\varepsilon_f(t)$ is the strain of the fibers and $\varepsilon_m(t)$ is the strain of the matrix.

Due to the arrangement in series of the elastic spring and viscous dashpot of the matrix, one has that

$$\sigma_m(t) = \sigma_{E_m}(t) = \sigma_{\eta_m}(t), \tag{3}$$

where $\sigma_{E_m}(t)$ and $\sigma_{\eta_m}(t)$ are the elastic and viscous stresses of the matrix, respectively. Furthermore, the strain of the matrix, $\varepsilon_m(t)$, is

$$\varepsilon_m(t) = \varepsilon_{E_m}(t) + \varepsilon_{\eta_m}(t), \tag{4}$$

where $\varepsilon_{E_m}(t)$ and $\varepsilon_{\eta_m}(t)$ are the strain of the spring and the strain of the viscous dashpot for the matrix, respectively.

The elastic stress of the matrix is defined as

$$\sigma_{E_m}(t) = E_m \varepsilon_{E_m}(t) \tag{5}$$

where E_m denotes the elastic modulus of the matrix. The viscous stress of the matrix is defined as

$$\sigma_{\eta_m}(t) = \eta_m \varepsilon'_{\eta_m}(t), \tag{6}$$

where η_m denotes the viscous modulus of the matrix and a prime denotes the differentiation with respect to t.

After noting that $\sigma_m(t) = \sigma_{E_m}(t)$ from Eq. (3) and that $\sigma_m(t) = \sigma(t) - \sigma_f(t)$ from Eq. (1), Eq. (5) can be rewritten as

$$\varepsilon_{E_m}(t) = \frac{\sigma(t) - \sigma_f(t)}{E_m}. \tag{7}$$

Moreover, since $\varepsilon_{\eta_m}(t) = \varepsilon_m(t) - \varepsilon_{E_m}(t)$ from Eq. (4) and $\varepsilon(t) = \varepsilon_m(t)$ from Eq. (2), Eq. (6) becomes

$$\sigma_{\eta_m}(t) = \eta_m \varepsilon'(t) - \eta_m \varepsilon'_{E_m}(t). \tag{8}$$

By recalling that $\sigma_m(t) = \sigma_{\eta_m}(t)$ from Eq. (3) and using Eq. (8), Eq. (1) takes the form

$$\sigma(t) = \sigma_f(t) + \eta_m \varepsilon'(t) - \eta_m \varepsilon'_{E_m}(t). \tag{9}$$

Finally, after computing $\varepsilon'_{E_m}(t)$ from Eq. (7) and substituting the resulting expression into Eq. (9) one obtains the governing equation for the system described in Fig. 1

$$\sigma'(t) + \frac{E_m}{\eta_m}\sigma(t) = \sigma_f'(t) + \frac{E_m}{\eta_m}\sigma_f(t) + E_m \varepsilon'(t). \tag{10}$$

The ratio η_m/E_m is a characteristic time, τ, usually called the *relaxation time*. The above governing equation can be then rewritten as

$$\sigma'(t) + \frac{\sigma(t)}{\tau} = \sigma_f'(t) + \frac{\sigma_f(t)}{\tau} + E_m \varepsilon'(t). \tag{11}$$

It must be noted that Eq. (11) is derived solely by considering the arrangement of the constituents of the tissue as depicted in Fig. 1 and the Maxwell-type viscoelastic behavior of the matrix defined in Eqs. (5) and (6). In other words, no assumption on the constitutive behavior of the collagen fibers and, thus, on the stress of the collagen fibers, $\sigma_f(t)$, has been made to derive Eq. (11). Once the stress of collagen fibers, $\sigma_f(t)$, is defined, the governing Eq. (11) with appropriate initial condition can be used to describe relaxation, creep, and strain stiffening phenomena.

2.2. Stress of collagen fibers

The stress of the fibrous component of the tissue, $\sigma_f(t)$, is defined by using a structural approach as previously done by other investigators [23–25]. The collagen fibers are assumed to become straight