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On the equivalence of two methods of determining fabric tensor

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ABSTRACT

In this paper it is studied how three methods of quantifying structural anisotropy are related. Mean intercept length (MIL) method has been designed for the analysis of binary images. Autocorrelation function and the covariance matrix of the gray-level intensity gradient (GST method) are approaches designed for the analysis of gray-level data. It is shown here that the autocorrelation function and the MIL methods are not related in a general case. In contrast, an analytical proof is provided to show that MIL and GST methods are strictly equivalent. The standard definition of MIL is expressed in terms of a gradient field. Next it is shown that eigenvectors of the MIL fabric tensor are also eigenvectors of the GST fabric tensor and eigenvalues of the MIL fabric tensor can be determined if the eigenvectors of the GST fabric tensor are known. It follows from the study that the application of MIL in the assessing quality of trabecular bone can be replaced in all cases by the application of the GST method, which is more general (as defined for gray-level data), easier to implement and less computationally expensive.

1. Introduction

Human bone is generally classified into either cortical (compact) or trabecular (cancellous or spongy) bone. All bones have an exterior layer (cortex) composed of cortical bone. In some parts of the interior of bones bony tissue may be arranged in a network of intersecting plates and rods called trabeculae. The intertrabecular space is filled with blood vessels and marrow. Trabecular bone is a highly porous structure with porosity ranging from 40% to even 95%. One of the most striking properties of trabecular bone is its structural anisotropy. Trabeculae are not aligned in random directions. Rather, in a representative volume element, they are averagely aligned parallel with the lines of major compressive or tensile stresses. The mechanical properties of trabecular bone are also anisotropic, so the first requirement for a formulation of a relation between mechanics and architecture must be an ability to quantify structural anisotropy. To quantify structural anisotropy of trabecular bone structure, Cowin [1] introduced the term "fabric tensor" and proposed equations relating fabric tensor and density to the elastic constants. Based on Cowin's ideas, it has been shown [2–4] that variations of structural anisotropy and bone volume fraction, derived from ultra high-resolution μCT images, explain at least 90% of the variation of the apparent elastic constants. These approaches have been however substantially based on binary 3D data and, consequently, methods of quantifying structural anisotropy applicable to binary images only have been used (primarily methods of mean intercept length (MIL) [5], volume orientation (VO) [6] or star volume distribution (SVD) [7]).

Clinical examinations of trabecular bone deliver low-resolution gray-level images, which cannot be easily converted to binary data without introducing serious segmentation artifacts. In fact, it is a well recognized problem that measures of structural anisotropy derived from low-resolution binary images, acquired under in vivo conditions do not correlate well with measures derived from high-resolution data [8,9] and are not as highly predictive of mechanical properties. On the other hand methods of quantifying fabric from gray-level data do exist [10–13]. Gray-level intensity gradient-based fabric tensor has been successfully used [10] to explain variation of Young's modulus of entire vertebral bodies.

The present study is inspired by the experimental findings reported in Ref. [10]. So far, except in Ref. [10], only binary-image-based methods of quantifying structural anisotropy were used to explain mechanical competence of trabecular bone. The successful explanation of the variation of the Young's modulus, using a gray-level method of estimating structural anisotropy, when combined with the equally successful application of binary-image-based methods for high-resolution data suggests, that there may exist some fundamental correspondence between binary and gray-level methods. In the present study the possibility of the existence of such a correspondence is tested.

Because of its widespread use, MIL method was selected in the present study as the reference approach (MIL is implemented also in commercially available analyzers of medical images e.g. Skyscan^{TM} CT-analyzer software, SkyScan, Belgium). The collected experimental results [10–13] suggest that one should consider

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autocorrelation and intensity gradient-based methods as potentially related to MIL. In the present study the equivalence of these methods is tested, using primarily analytical tools. It is shown that autocorrelation and MIL methods are not related in a general case. In contrast, an analytical proof is provided in 2D that the MIL and gradient of gray-level intensity fabric tensors are strictly equivalent. Strong numerical evidence suggests that the same is true in 3D. Because computing fabric tensor from the gray-level gradient is both simpler in implementation and less computationally expensive, it follows from the presented results that MIL method can be replaced with gained performance by the equivalent gradient of the gray-level intensity fabric tensor.

2. Problem formulation

Formally, structural anisotropy (fabric) is a second rank, positive definite tensor describing the anisotropy of the mass distribution within a porous material. There are a few approaches, referenced below, to make this definition more specific. The most straightforward approach requires defining a vector field, which specifies locally the structure orientation. This approach is discussed in the following.

Let **V** be a unit vector describing the mean orientation of trabeculae within a trabecular structure and $\mathbf{g}(x,y,z)$ be a (not necessarily unit) vector describing local orientation of trabeculae. The error vector $\mathbf{e}(x,y,z)$ of **g** with respect to **V** is equal to $\mathbf{e}(x,y,z) = \mathbf{g} - (\mathbf{g}^{\mathrm{T}} \cdot \mathbf{V})\mathbf{V}$. The total error E is equal to [14]:

$$E = \int_{\Omega} |\boldsymbol{e}|^2 \, \mathrm{d}\varpi \tag{1}$$

where the integration is performed over some region of interest Ω. The orientation vector **V** can be found by minimizing E with respect to **V**. Because **V** was constrained to be a unit vector, one has to minimize the following quantity:

$$E' = \int_{\Omega} |\boldsymbol{e}|^2 \, \mathrm{d}\varpi + \lambda(|\boldsymbol{V}|^2 - 1) \tag{2}$$

where λ is the Lagrange multiplier. Recalling that $|\mathbf{V}|^2 = \mathbf{V}^{\mathrm{T}} \cdot \mathbf{V}$, Eq. (2) can be rewritten in the following form:

$$E' = \int_{\Omega} (\boldsymbol{g}^{\mathrm{T}} \cdot \boldsymbol{g} - (\boldsymbol{g}^{\mathrm{T}} \cdot \boldsymbol{V})^2 (1 - \boldsymbol{V}^{\mathrm{T}} \cdot \boldsymbol{V}) \\ - (\boldsymbol{g}^{\mathrm{T}} \cdot \boldsymbol{V})(\boldsymbol{V}^{\mathrm{T}} \cdot \boldsymbol{g})) \, \mathrm{d}\varpi + \lambda(\boldsymbol{V}^{\mathrm{T}} \cdot \boldsymbol{V} - 1) \tag{3}$$

The first term in the integral it is a constant, the second term is zero by assumption (the length of **V** is equal to 1). Hence one has:

$$E' = \mathrm{const} - \int_{\Omega} (\boldsymbol{g}^{\mathrm{T}} \cdot \boldsymbol{V})(\boldsymbol{V}^{\mathrm{T}} \cdot \boldsymbol{g}) \, \mathrm{d}\varpi + \lambda(\boldsymbol{V}^{\mathrm{T}} \cdot \boldsymbol{V} - 1) \\ = \mathrm{const} - \sum_{i,j} \int_{\Omega} g_i g_j V_i V_j \, \mathrm{d}\varpi + \lambda \sum_i (V_i V_i - 1) \tag{4}$$

Differentiation of E' with respect to the mth component V_m of **V** gives the following equation for **V**:

$$\frac{\partial E'}{\partial V_m} = 0 = -\int_{\Omega} g_m g_j V_j \, \mathrm{d}\omega + \lambda V_m \tag{5}$$

The integrals $\mu_{i,j} = \int_{\Omega} g_i g_j \, \mathrm{d}\omega$ are the components of the tensor $\boldsymbol{\mu}$ of structural anisotropy, related to the vector field **g**. The mean orientation vector **V** is the solution of the eigen equation $\boldsymbol{\mu}\mathbf{V} = \lambda\mathbf{V}$.

One of the typical choices of the local orientation vector field is performed on the base of the volume orientation (VO) method [6] (Fig. 1). The method works for binary images of binary structures. A local volume orientation is defined at any point within a trabecula as the orientation of the longest intercept through that point. For every point within the analyzed structure a single unit vector describing the local orientation is thus obtained. Then the components of this vector are inserted into the integrals $\mu_{i,j}$ (or a summations in a discrete case), defining the VO fabric tensor.

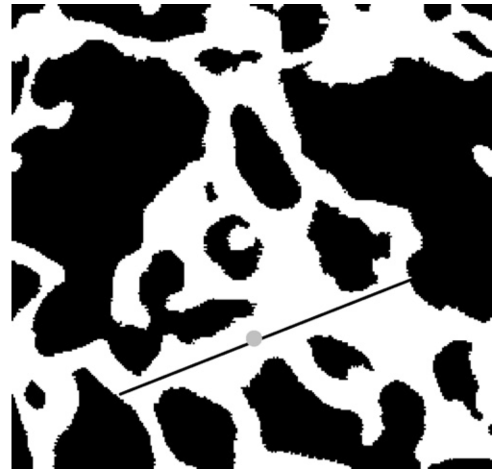

Fig. 1. Measurement of the volume orientation in a sample point (gray disk).

Star length distribution [15] (SLD), like the volume orientation, describes the typical distribution of trabecular bone around a typical point within a trabecula. To calculate the SLD fabric tensor, lines along specified directions uniformly distributed within a unit sphere and emerging from a given seed point n are traced until an interception with the bone-marrow interface is found. The length l_n of the intercept line from the seed point n to the boundary is recorder for every direction. As the result of the measurements performed at a single point a set of vectors with varying lengths and orientations is obtained. All these vectors are inserted into integrations (or summations) defining the SLD fabric tensor. Star volume distribution [7] (SVD) is a modification of the SLD method—the vectors, which are inserted into integration, defining the SVD fabric tensor are weighted with l_n^3, but not with l_n, like for the SLD method.

Another classical approach to characterizing structural anisotropy is based on the mean intercept length (MIL) method [5]. The principle of the MIL measurement is to count the number of intersections between a family of equidistant parallel lines and the bone/marrow interface as the function of the 3D orientation θ of the family of lines (see Fig. 2 for a 2D example). Decreasing the inter-line distance and increasing the number of orientation for which the number of intersections is counted improves precision of MIL estimation at the cost of increased computational burden. There are however no rules of thumb which specify how these parameters should be set. Ideally, the inter-line distance should be

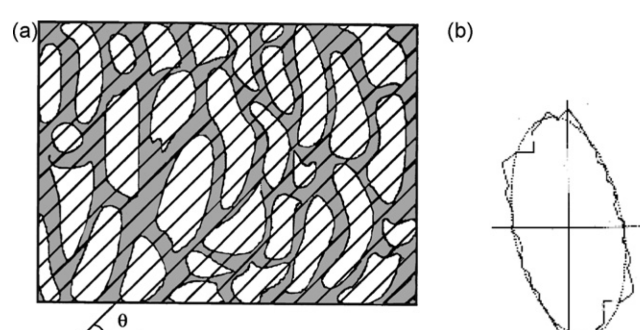

Fig. 2. The principles of the MIL measurement: (a) a linear grid imposed onto the structure and (b) MIL is a function of orientation θ.