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The application of subspace preconditioned LSQR algorithm for solving the electrocardiography inverse problem

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ABSTRACT

Regularization is an effective method for the solution of ill-posed ECG inverse problems, such as computing epicardial potentials from body surface potentials. The aim of this work was to explore more robust regularization-based solutions through the application of subspace preconditioned LSQR (SP-LSQR) to the study of model-based ECG inverse problems. Here, we presented three different subspace splitting methods, i.e., SVD, wavelet transform and cosine transform schemes, to the design of the preconditioners for ill-posed problems, and to evaluate the performance of algorithms using a realistic heart-torso model simulation protocol. The results demonstrated that when compared with the LSQR, LSQR-Tik and Tik-LSQR method, the SP-LSQR produced higher efficiency and reconstructed more accurate epicardial potential distributions. Amongst the three applied subspace splitting schemes, the SVD-based preconditioner yielded the best convergence rate and outperformed the other two in seeking the inverse solutions. Moreover, when optimized by the genetic algorithms (GA), the performances of SP-LSQR method were enhanced. The results from this investigation suggested that the SP-LSQR was a useful regularization technique for cardiac inverse problems.

1. Introduction

The electrocardiography (ECG) inverse problem aims to quantitatively reconstruct epicardial, endocardial and myocardial potentials, as well as electrograms and isochrones of the heart from torso surface potentials [1–4]. Body surface potentials (BSPs, φ_B) are related to epicardial potentials (EPs, φ_H) through the following linear system equation:

$$\boldsymbol{A}\varphi_H = \varphi_B \tag{1}$$

where $\boldsymbol{A}$ is the transfer matrix associated with volume conductor (torso) properties including geometry, conductivity and distance between epicardial surface nodes and the torso surface nodes.

It is known that the system function (1) is ill-posed [5] and it is generally solved with regularization approaches, such as Tikhonov regularization [6,7] and truncated singular-value decomposition (TSVD) [8]. And several L-curve [9,10], generalized cross-validation (GCV) [11] and zero crossing schemes [12], have been developed for the determination of optimal regularization parameters and corresponding inverse solutions.

In addition to direct regularization schemes, we also used iterative methods [13,14] for the solution of ill-posed problem. For iterative algorithms, convergence was very important and preconditioning functions as an effective way to enhance the convergence rate of the algorithms. Hanke and Vogel [15] proposed an approach for solving the Tikhonov problem with suitable preconditioning, which was used for solving the least squares formulation of the Tikhonov problem:

$$\begin{aligned}\varphi_H(\lambda) &= \arg\min_{\varphi_H}||\hat{\boldsymbol{A}}\varphi_H - \hat{\varphi}_B||_2 = \arg\min_{\varphi_H}\left\|\begin{matrix}\boldsymbol{A}\varphi_H - \varphi_B \\ \lambda^2 I\varphi_B\end{matrix}\right\|_2^2 \\ &= \arg\min_{\varphi_H}\left\{\sum_i(\boldsymbol{A}\varphi_H - \varphi_B)_i^2 + \lambda^2\sum_i\varphi_{Bi}^2\right\} \\ &= \arg\min_{\varphi_H}\{||\boldsymbol{A}\varphi_H - \varphi_B||_2^2 + \lambda^2||\varphi_B||_2^2\}\end{aligned} \tag{2}$$

where $\hat{\boldsymbol{A}} = \begin{bmatrix}\boldsymbol{A} \\ \lambda^2 I\end{bmatrix}$ and $\hat{\varphi}_B = \begin{bmatrix}\varphi_B \\ 0\end{bmatrix}$.

The key point of the algorithm was to divide the original inverse solution space into two subspaces, one of them with a smaller dimension. Proper basis vectors were chosen so that the smaller subspace could be used for the representation of the regularized solutions. Another implementation method (Subspace

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Preconditioned LSQR algorithm, SP-LSQR) for solving the two-level Schur complement system was presented by Jacobsen and Hansen [16,17]. The method was more efficient and more robust than the two-level Schur complement CG algorithm. Later, a modified version of the two-level iterative method [18] was proposed, which showed a similar numerical performance.

Recently, to search robust regularization techniques for cardiac inverse problems, we have investigated a series of regularization schemes [4,14,19] and in this work the iterative SP-LSQR algorithms were paid particular attention, with the intention of having a comprehensive set of regularization tools for the ECG inverse studies.

2. Methods

2.1. The subspace LSQR algorithm

In the subspace LSQR algorithm, the two-level preconditioners were designed to solve the Tikhonov problem. The regularized solution $\varphi_{H\lambda}$ could be obtained in the form:

$$\varphi_{H\lambda} = \phi u + \psi v \tag{3}$$

where $\phi \in \Re^{n\times k}$ and $\psi \in \Re^{n\times(n-k)}$. Here $[\phi,\Psi]$ was an orthogonal basis of $\Re^n$, and the selection of basis vectors were usually selected from the anterior columns to ensure that they contained low frequency information. Furthermore, the number of columns of ϕ needed to be small, to avoid introducing small singular values into the first part (ϕu), which could sometimes disturb the inverse solutions. Both requirements lead to the fact that the most important part of $\phi_{H\lambda}$ was ϕu and therefore it could be easily calculated by a direct method.

When substituting the regularized inverse solutions of the form (3) into the Tikhonov problem, the problem could be reformatted to be the following minimization problem:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \arg\min_{u,v} \left\| \hat{\boldsymbol{A}} \begin{bmatrix} \phi & \psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \hat{\varphi}_B \right\|_2 \tag{4}$$

the QR factorization of the matrix could be denoted as follows:

$$\hat{\boldsymbol{A}}\phi = \begin{bmatrix} \boldsymbol{A} \\ \lambda^2 I \end{bmatrix} \phi = Q \begin{bmatrix} R \\ 0 \end{bmatrix} = \begin{bmatrix} Y & Z \end{bmatrix} \begin{bmatrix} R \\ 0 \end{bmatrix} = YR \tag{5}$$

with Q orthogonal and R upper triangular, then we would have:

$$\begin{aligned} \begin{bmatrix} u \\ v \end{bmatrix} &= \arg\min_{u,v} \left\| Q^T \hat{\boldsymbol{A}} [\, \phi \quad \psi \,] \begin{bmatrix} u \\ v \end{bmatrix} - Q^T \hat{\varphi}_B \right\|_2 \\ &= \arg\min_{u,v} \left\| [\, Y \quad Z \,]^T \hat{\boldsymbol{A}} [\, \phi \quad \psi \,] \begin{bmatrix} u \\ v \end{bmatrix} - [\, Y \quad Z \,]^T \hat{\varphi}_B \right\|_2 \\ &= \arg\min_{u,v} \left\| \begin{bmatrix} Y^T\hat{\boldsymbol{A}}\phi & Y^T\hat{\boldsymbol{A}}\psi \\ Z^T\hat{\boldsymbol{A}}\phi & Z^T\hat{\boldsymbol{A}}\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} Y^T\hat{\varphi}_B \\ Z^T\hat{\varphi}_B \end{bmatrix} \right\|_2 \\ &= \arg\min_{u,v} \left\| \begin{bmatrix} R & Y^T\hat{\boldsymbol{A}}\psi \\ 0 & Z^T\hat{\boldsymbol{A}}\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} Y^T\hat{\varphi}_B \\ Z^T\hat{\varphi}_B \end{bmatrix} \right\|_2 \end{aligned} \tag{6}$$

Thus, the inverse solutions $\varphi_{H\lambda}$ could be solved in the form:

$$\begin{cases} v = \arg\min_{v} ||Z^T\hat{\boldsymbol{A}}\psi v - Z^T\hat{\varphi}_B||_2 & (a) \\ Ru = Y^T(\hat{\varphi}_B - \hat{\boldsymbol{A}}\psi v) & (b) \\ \varphi_{H\lambda} = \phi u + \psi v & (c) \end{cases} \tag{7}$$

After defining a variable $p = \Psi v$, the equation could be rewritten as follows:

$$\begin{cases} p = \arg\min_{p} ||Z^T\hat{\boldsymbol{A}}p - Z^T\hat{\varphi}_B||_2 & (a) \\ Ru = Y^T(\hat{\varphi}_B - \hat{\boldsymbol{A}}p) & (b) \\ \varphi_{H\lambda} = \phi u + \psi p & (c) \end{cases} \tag{8}$$

In Eq. (8.a), the variable p could be calculated by the LSQR iterative method and the Eq. (8.b) could be solved directly with the known p. In this way, the inverse solutions $\varphi_{H\lambda}$ could be obtained by (8.c).

The right singular vectors associated with the first kth largest singular values of the matrix $\boldsymbol{A}$, were often implemented as the optimal selection of subspace [16,18]. However, an optimal subspace could not always be produced by the singular-value decomposition (SVD) method. Therefore other algorithms such as the wavelet transform [20] and the cosine transform [21] have been considered as alternative bases $[\phi,\Psi]$ for seeking such a subspace. In this work, two transforms have been considered for our study. In the wavelet transformation, the Daubechies wavelets were used (where $D=4$), and the wavelet package developed by Nielsen [22] was introduced to implement the wavelet transform. The Discrete cosine transform (DCT) was conducted using the Matlab build-in routine. Both transforms met the condition set, that is, the basis vectors had a higher frequency as the column index grew. In this work, the performances of these three different subspace splitting methods were evaluated in solving the ECG inverse problem.

2.2. Simulation protocol

The simulation protocol was based on a geometrically realistic heart-torso model [14,23,24] as shown in Fig. 1(a) which depicted the epicardial, lung and torso geometries and corresponding mesh information, where: the epicardial model: 187 nodes and 346 triangles; lung model: 297 nodes and 586 triangles; torso model: 412 nodes and 820 triangles. The transfer matrix $\boldsymbol{A}$ was obtained by the Boundary Element Method (BEM). In this study, we considered 220 torso surface nodes distributed in the vicinity of the heart (Fig. 1(a), marked with a red star), which represented the electrode sites in an assumed clinical body surface mapping system. For the inverse problem, the system matrix had a dimension of 220×187 which was obtained by an extraction procedure (from 412×187 down to 220×187), the resulting condition number of the system matrix being 2.25×10^{12}. Fig. 1(b) illustrated the different action potentials (APs) for various myocardial cells [25] and the normal ventricular excitation sequence [23], from which the transmembrane APs (φ_m) could be calculated. A heart surface source method [26,27] was used to calculate the EPs for the normal heart, which related the whole heart surface potential φ_{WH} to transmembrane APs (φ_m) as the following equation:

$$\varphi_{WH} = T_{mH}\varphi_m \tag{9}$$

Fig. 1(c) shows the closet surface that bounds the heart, including epicardial surface (187 nodes and 346 triangles), endocardial surface (290 nodes and 440 triangles), and top surface (21 nodes and 106 triangles). Following the heart surface source method, the relation matrix T_{mH} could be calculated from the closest heart surface geometry information. Thus the whole heart surface potential φ_{WH} could be obtained (see in Fig. 1(d)), and the EPs φ_H (Fig. 1(e)) could be used as the source to calculate the BSPs φ_B (Fig. 1(f)). Here, a normal ventricular excitation sequence of the time at 27 ms, after the onset of activation in the ventricles, is illustrated for the calculation of epicardial potentials and body surface potentials.

In the inverse problem study, the torso surface potentials and the transfer matrix $\boldsymbol{A}$ were employed to seek the EPs. To simulate the noise involved in clinical practice, the linear system equation