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A spatial frequency spectral peakedness model predicts discrimination performance of regularity in dot patterns

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ABSTRACT

Subjective assessments of spatial regularity are common in everyday life and also in science, for example in developmental biology. It has recently been shown that regularity is an adaptable visual dimension. It was proposed that regularity is coded via the peakedness of the distribution of neural responses across receptive field size. Here, we test this proposal for jittered square lattices of dots. We examine whether discriminability correlates with a simple peakedness measure across different presentation conditions (dot number, size, and average spacing). Using a filter-rectify-filter model, we determined responses across scale. Consistently, two peaks are present: a lower frequency peak corresponding to the dot spacing of the regular pattern and a higher frequency peak corresponding to the pattern element (dot). We define the "peakedness" of a particular presentation condition as the relative heights of these two peaks for a perfectly regular pattern constructed using the corresponding dot size, number and spacing. We conducted two psychophysical experiments in which observers judged relative regularity in a 2-alternative forced-choice task. In the first experiment we used a single reference pattern of intermediate regularity and, in the second, Thurstonian scaling of patterns covering the entire range of regularity. In both experiments discriminability was highly correlated with peakedness for a wide range of presentation conditions. This supports the hypothesis that regularity is coded via peakedness of the distribution of responses across scale.

1. Introduction

Regular spatial patterns appear in natural and artificial systems at a wide range of scales. Although not always defined, regularity can be regarded as a simple law that governs the appearance of an image. There exist distinct types of regularity and these may depend on the specific features of the image. For example, we frequently encounter patterns with repeating elements placed at equal spacings. This type of pattern is defined by the set of element locations (called the point pattern), and the form of the individual elements placed at each point (e.g., dots, as used here). Such an arrangement can be described by a straightforward law of periodicity according to which, neglecting edge effects, an image, I, appears identical to itself, when it is translated by an integer number, m, of a quantized step, $\vec{d}$, in one or more directions (i.e., $I(\vec{x} + m\vec{d}) = I(\vec{x})$). Similar invariance laws can also describe reflection or rotational symmetries and are well studied (Miller, 1972; O'Keeffe & Hyde, 1996; Griffin, 2009). Vision strongly engages with regularity even when the underlying law is not identified or cognitively accessible as, for example, in Glass patterns (Glass, 1969) or in patterns with self-similarity at multiple scales. Specific types of symmetry have been appreciated and used historically in architecture and arts long before their explicit mathematical formulation was derived.

Regularities may interact synergistically (Wagemans, Wichmann, & Op de Beeck, 2005) and in a generally unpredictable fashion. For example, when the horizontal distance between dots in a square lattice decreases, this can give rise to a new percept: the appearance of notional vertical lines (Wagemans, Eycken, Claessens, & Kubovy, 1999). Regularity can cause pop-out effects and can be considered a type of Gestalt (Koffka, 1935; Ouhnana, Bell, Solomon, & Kingdom, 2013). Attneave (1954) considered the ability of the visual system to detect regularity as a mechanism of the perceptual machinery to reduce redundancy by compressing information and thus increase coding

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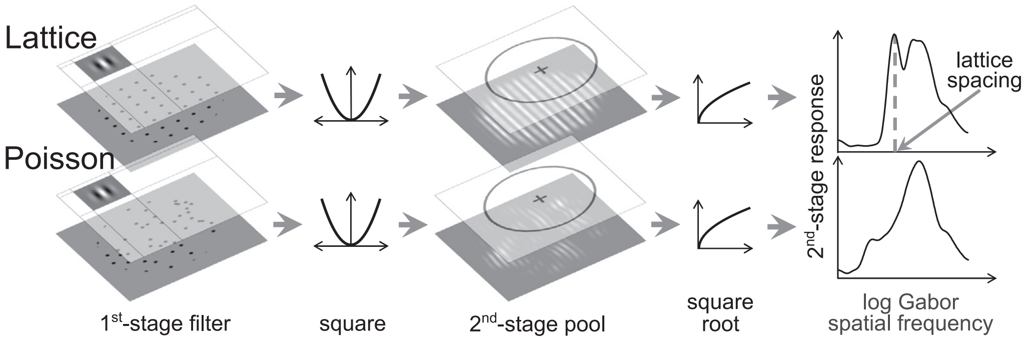

Fig. 1. Demonstration of the filter-rectify-filter process for two dot patterns. The top row demonstrates the process for a perfectly regular pattern and the bottom row for a Poisson pattern. The dot pattern was convolved with a series of Gabor filters of varying spatial scale. The outputs were squared and pooled across a fixed circular area and then a final square root nonlinearity was applied. Comparing the two spectra, one can see the narrow peak that corresponds to the lattice spacing in the upper-right graph; this peak is absent for the Poisson pattern.

efficiency. In nature, perfect regularity is rare. The visual system most often deals with partial regularity, i.e., some amount of departure from perfect regularity. In textures, the degree of regularity is a cue for texture discrimination and segmentation (Bonneh, Reisfeld, & Yeshurun, 1994; Vancleef et al., 2013). Regularity also interacts with other perceptual dimensions, e.g., numerosity (Whalen, Gallistel, & Gelman, 1999) and needs to be controlled in psychophysical experiments (Allïk & Tuulmets, 1991; Bertamini, Zito, Scott-Samuel, & Hulleman, 2016; Burgess & Barlow, 1983; Cousins & Ginsburg, 1983; Ginsburg, 1976, 1980; Ginsburg & Goldstein, 1987). Similarly, in contour-integration tasks, stimuli of intermediate regularity must be used to avoid density cues (Demeyer & Machilsen, 2012; Machilsen, Wagemans, & Demeyer, 2015).

Perception of partial regularity is useful for scientific analysis. Researchers very often rely on vision to assess the degree of organization in patterns encountered in the study of evolving systems. Partial regularity is essential in natural sciences. In biological organisms, high regularity is advantageous as it affects efficiency (e.g., in the eye it allows for a high density of receptors at the fovea), while lack of regularity manifests as disease (e.g., cancer) and compromised homeostasis. In some processes, however, what is crucial is the balance between perfect and partial regularity. For example, during development, dynamic noise keeps tissue in a state of intermediate regularity, protecting cell proliferation by maintaining a dynamic equilibrium between newly generated cells with division processes and cell death. In this way, biological functions are able to adjust to changes and so exhibit robustness across different developmental conditions (Cohen, Baum, & Miodownik, 2011; Cohen, Georgiou, Stevenson, Miodownik, & Baum, 2010; Marinari et al., 2012). Interestingly, despite its importance, there is no unified framework for estimation of the degree of regularity. Rather, there are a variety of isolated approaches (e.g., Cliffe & Goodwin, 2013; Dunleavy, Wiesner, & Royall, 2012; Jiao et al., 2014; Sausset & Levine, 2011; Steinhardt, Nelson, & Ronchetti, 1983; Truskett, Torquato, & Debenedetti, 2000). Occasionally, researchers are hesitant to trust measures they use, as they report an obvious disagreement between the measure and what they perceive visually when examining the organization of a system (Cook, 2004). Humans are particularly consistent in their judgments of regularity even for diverse sets of stimuli (Protonotarios, Baum, Johnston, Hunter, & Griffin, 2014; Protonotarios, Johnston, & Griffin, 2016), and since these judgments have an interval-scale structure (Stevens, 1946), they can be used as a basis for quantification. By analyzing the process of pattern formation in the developing *Drosophila* epithelium, it has been demonstrated that an objective surrogate of perceived regularity can be used for scientific analysis (Protonotarios et al., 2014).

Regularity is thus an important aspect of stimuli for the visual system. However, little is known about how it is encoded in the brain. Ouhnana et al. (2013) showed that regularity is an adaptable visual dimension, and proposed that it is coded via the peakedness of the distribution of neural responses across receptive-field size. They used patterns consisting of luminance-defined (Gaussian blobs), and contrast-defined (difference of Gaussians and random binary patterns) elements arranged on a square grid, and they varied the degree of regularity by randomly jittering their position. It was found that a test pattern appears less regular after adaptation to a pattern of similar or higher degree of regularity. The strength of this uni-directional aftereffect was dependent on the degree of regularity of the adapting pattern, with higher regularity causing a stronger effect. Based on the observation that the amplitude of the Fourier transformation of a regular pattern is also regular, they suggested that regularity information is carried mainly by the amplitude spectrum and not the phase. They proposed that regularity is coded via the pattern of response amplitudes of visual filters of varying receptive-field size and that adaptation alters this pattern of responses.

To illustrate the point, they simulated a simple filter-rectify-filter model of neural responses (Graham, 2011) and examined the distribution of responses across scale for a perfectly regular and a random-dot pattern. The sequence of processing stages is illustrated in Fig. 1. First, a bank of bandpass filters of varying size is applied to the image. Here, the receptive fields are vertical Gabors:

$$F(x, y) = exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)cos(2\pi fx), \tag{1}$$

with the standard deviation of the Gaussian envelope, σ, varying to cover a range of spatial scales. σ covaried with spatial frequency, f, according to:

$$\sigma = \frac{1}{\pi f}\sqrt{\frac{ln2}{2}}\frac{2^b + 1}{2^b - 1} \tag{2}$$

to maintain a constant full-width, half-height spatial frequency bandwidth, b (in octaves). The Gabor filter is normalized by a factor of σ^2 to keep energy sensitivity constant across scale. The responses of the first-stage filter are then rectified by squaring and a second-stage low-pass filter sums the output over a large region and takes the square root. Fig. 1 illustrates the output of the filter-rectify-filter cascade across scale for a regular and an irregular dot pattern. In the spectrum of the regular pattern two predominant peaks can be seen. The first appears at lower spatial frequency and coincides with the lattice spacing, while a second broader peak at higher frequency corresponds to the pattern element (dot) size. The first peak is absent in the irregular pattern. Although this analysis is applied to patterns of luminance-defined elements, it can be easily generalized to contrast-defined elements by the introduction of an additional intermediate stage of bandpass filters and non-linearity (Ouhnana et al., 2013). Ouhnana et al. (2013) suggested that regularity is coded via some measure of peakedness of this distribution.

We can use data from previous work to verify that this is a reasonable assumption. We examined the perception of regularity for dot patterns that were based on a square lattice (Protonotarios et al., 2016). Regularity was varied using positional jitter. We used patterns that covered the whole range of regularity, from a perfect lattice to total