Journal of Economic Behavior & Organization 109 (2015) 1–9

Contents lists available at ScienceDirect

Journal of Economic Behavior & Organization

journal homepage: www.elsevier.com/locate/jebo

# On divergent dynamics with ordinary least squares learning☆

Shurojit Chatterji[a], Ignacio N. Lobato[b,*]

[a] School of Economics, Singapore Management University, Singapore
[b] Centro de Investigación Económica, ITAM, Ave. Camino Santa Teresa 930, Mexico, DF 10700, Mexico

ARTICLE INFO

*Article history:*
Received 22 May 2013
Received in revised form 13 February 2014
Accepted 16 October 2014
Available online 4 November 2014

*Keywords:*
Temporary equilibrium
Ordinary least squares learning
Stability

ABSTRACT

This article addresses the stability properties of a simple economy (characterized by a one-dimensional state variable) when the representative agent, confronted by trajectories that are divergent from the steady state, performs transformations in that variable in order to improve forecasts. We find that instability continues to be a robust outcome for transformations such as differencing and detrending the data, the two most typical approaches in econometrics to handle nonstationary time series data. We also find that inverting the data, a transformation that can be motivated by the agent reversing the time direction in an attempt to improve her forecasts, may lead the dynamics to a perfect-foresight path.



## 1. Introduction

The literature on learning in economic models has focussed on identifying conditions under which the dynamics converge to self-fulfilling expectations. This literature shows that under plausible conditions, learning dynamics are actually divergent.[1] Along such paths it becomes hard to justify why agents should stick to their learning model. One may argue that along paths where forecasts appear not to improve over time, agents ought to question the validity of their learning model and try alternate specifications. Whether such behaviour would lead to more "stable" dynamics with better forecasts, is a question that appears to be relevant to the broader issue of expectation formation in decentralized environments.

Motivated by such concerns, in a recent article (Chatterji and Lobato, 2010, CL henceforth) studied the dynamics of simple economic models where, in an attempt to improve her forecasts, a representative agent transforms the state variable before performing ordinary least squares (OLS, henceforth) learning. Following the practice in econometrics of transforming the data before the analysis, CL initiated the analysis of the stability effects[2] of transformations of the state variable and showed that the OLS learning dynamics depend crucially on the type of (instantaneous) transformation of the state variable that the agents forecast. In particular, CL emphasized the destabilizing effect that concave transformations, which are typically the ones an agent would use upon observing divergent data, such as taking logarithms, may have. However, CL restricts attention to some classes of instantaneous transformations of the state variable. Whereas static transformations only employ the

☆ Chatterji acknowledges research support from SMU grant no. 10-C244-SMU-010. Lobato acknowledges financial support from the Mexican CONACYT, reference number 151264, and from Asociación Mexicana de Cultura. We are grateful to a referee for very helpful comments.

* Corresponding author. Tel.: +52 55 56284191.

[1] It is well known (Grandmont, 1998) that OLS learning, since it extrapolates all trends in past data, leads to locally unstable dynamics.

[2] It is worth emphasizing that in the setting of this paper, as in CL (2010) and Grandmont (1998), one does not impose on the representative agent a desire to stabilize the macro dynamics. In this literature an individual agent is treated as an infinitesimal who believes she is unable to influence the stability of the system, and her only concern therefore is for improvements in her forecasts rather than on stabilizing the system. However, agents may do so in a way that collectively destabilizes the steady-state.

http://dx.doi.org/10.1016/j.jebo.2014.10.003
0167-2681/

current value of the state variable, dynamic transformations may be more suitable in learning contexts where the agents are concerned with the evolution of the economy, since such transformations use current and past values of the state variable.

This article continues this research programme and extends CL in three directions. First, we start by analyzing dynamic transformations, second, we consider a specific instantaneous transformation (inverting the data) that was not covered in CL, and third, we allow a more general equilibrium map that also includes the past value of the state variable. Our focus here is on divergent trajectories and our concern is with the agent being able to learn the rate at which the state variable adjusts across successive periods. Instability (stability) in this paper will refer to trajectories where the agent is unable (able) to learn the rate of adjustment, whereas in CL, the focus being on the stability of the steady-state, instability referred to trajectories where the state variable is unbounded. The specific dynamic transformations that we study are motivated by standard econometric practice. When the agent perceives that the economy is not in a stationary environment, she may attempt the same data transformations an econometrician would. In this environment econometricians have typically employed two dynamic transformations to achieve a stationary framework. The main difference between both approaches is whether the agent (the econometrician) believes that the state variable is growing at a constant fixed rate or she believes that the state variable growth rate is random.

The first approach is called "trend-stationarity" in econometrics and implies that the agent believes that the state variable is stationary around a deterministic trend. In the simplest framework where the deterministic trend is linear, this model entails that the agent believes that the state variable has a constant growth rate, so that in order to achieve stationarity the agent should just linearly detrend the data. In case the agent believes the trend is a polynomial of higher order, stationarity requires that one subtract from the data a polynomial trend. Note that an econometrician, and similarly an agent, would continue to employ OLS as an appropriate estimation procedure in the presence of trends, as long as these trends satisfy Grenander's conditions (see Grenander and Rosenblatt, 1957), which allow for many types of trends. The second approach is called "difference-stationarity" in econometrics and it means that the agent believes that the state variable has an stochastic stationary growth rate. In this case it is said that the state variable has a unit-root, and, in order to achieve stationarity the agent should difference the (possibly logged) state variable data.

This article considers both scenarios, and shows that for an economy with feedback, that is, where the actual motion of the economy depends on the agent's beliefs about the evolution of the economy (which the Walrasian agent is assumed not to recognize), neither differencing nor detrending will in general help the agent to learn properly the economy's dynamics. A similar conclusion holds for the case of log differencing. Hence this article reinforces the message in CL that agents, who in order to improve their forecasts transform the state variable using standard dynamic transformations, will not succeed. Finally, we consider the case where the agent employs a natural, although uncommon in econometrics, transformation of the data, namely, inverting the data. This transformation, which was not covered in CL, is motivated by an economic agent who in the presence of instability prefers to consider a backwards estimation approach rather than the standard forward approach. Interestingly, we found that this transformation can lead the agent to a perfect-foresight path where the state-variable may diverge but forecasts are correct. The predetermined variable in the equilibrium map is important in ensuring the convergence to a perfect-foresight path where the state variable diverges at a finite rate.

Our work is related to a relatively small literature that studies the issue of model specification in the learning framework. Bray and Savin (1986) considered the case where agents question the validity of their OLS learning model (where unlike our setting, agents regress on a well-behaved exogenous process) using the Durbin–Watson first order serial correlation test. Bullard and Duffie (1998) examine dynamics using computational experiments in a general equilibrium model with heterogenous agents where agents learn by emulation and in particular experiment with new specifications of forecasting rules. Georges (2008), Section 8notes that when confronted with instability, it is reasonable to consider that sophisticated agents will experiment with a variety of functional forms and test parameter estimates for statistical significance. As an illustration, the author simulates an economy where agents consider switching between a simple forecasting rule based on sample averages and a more sophisticated AR(2) forecast rule based on the $t$-test. Cho et al. (2011) introduce a sequential model validation procedure and show that in the limit, the best *dominant* model is selected among a fixed set of models in the context of an economy with feedback.

The plan of the article is the following. Section 2 introduces the basic framework and identifies the divergence of learning dynamics. Section 3 examines the effects of the typical econometric transformations, whereas Section 4 analyzes the case of inverting the data. Section 5 briefly concludes. Appendix A contains the proofs and shows our instability conclusion remain valid for the common econometric practice of log-differencing, which involves a combination of dynamic and static transformations, in an alternative specification of the model where the state variable satisfies a positivity constraint.

## 2. The model

We consider the dynamics of an economy with a one dimensional state-variable whose value at date $t$ is denoted as $x_t$. The state-variable is expressed as a deviation from its steady state value, which is assumed to be known to the representative agent[3] (the agent, henceforth). The state-variable can take any value in the real line. The market clearing value of the state

[3] Since we study an economy where all agents are infinitesimal and have homogenous expectations, it is more convenient to assume a representative agent.