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Minimal two-way flow networks with small decay☆

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ABSTRACT

Information decay in networks generates two effects. First, it differentiates how well informed different players within the same component are, and therefore how attractive they are to sponsor links to. Second, players may prefer to sponsor links to players they are already connected to. By focusing on small decay we analyze the first effect in isolation. We characterize the set of Nash equilibrium networks in the two-way flow model of network formation with small decay for any increasing benefit function of the players. The results show that small decay is consistent with two well-known stylized facts, namely that (i) many real world networks have high diameters, and (ii) that the diameter of such networks is typically small relative to the population size. We show that even stochastically stable networks may have any diameter when the benefit function is linear or strictly concave. Finally we study implied stability relations. We find that if any non-empty minimal network is stable, then so is the periphery-sponsored star. With strictly convex benefit functions, we find that other stars tend to be stable for a larger range of parameters than larger diameter networks which satisfy our characterization. However, with strictly concave benefit functions the other stars are stable for a smaller range of parameters than the larger diameter networks.

1. Introduction

One of the key roles of socio-professional networks is the exchange of information. Exchanging information allows people to find fitting jobs,[1] recognize business opportunities,[2] and learn about new products.[3] It is therefore no surprise

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[1] Classical references are for instance Myers and Shultz (1951) and Granovetter (1974). See also Montgomery (1991).

[2] For instance, Shane and Cable (2002) show that social networks facilitate the acquisition of external capital for new ventures. They report that it is information gathering rather than social obligation which drives the result.

[3] Coleman et al. (1966) report how the diffusion of new medical drugs is facilitated by the social-professional network of physicians. See also Jackson (2008) for a good textbook on social and economic networks, and many interesting examples.

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0167-2681/

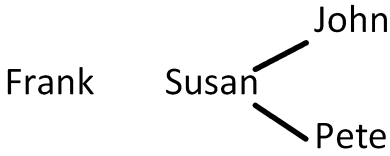

Fig. 1. Example network.

that people invest in their personal network. Networking events are common. People visit conferences;[4] businessmen join trade missions; and many people join network sites such as LinkedIn and Facebook.

An important feature of information networks is that acquired information can be passed on to others. Suppose John and Pete are not directly linked to each other, but are both directly connected to Susan (see Fig. 1). Then Susan may pass on relevant information from Pete to John. In this way information flows through networks. However, for several reasons John may benefit from being directly connected to Pete, rather than indirectly. First, Pete's information may be less timely if it has to pass through Susan before it reaches John. Second, communication is typically noisy. If information is passed on via others, in this case Susan, it accumulates more noise than if Pete provides it to John directly. Lastly, any person who passes on information may forget to pass on some part of it. Such processes are captured by the idea of 'information decay': information that is passed on more often before it reaches him is worth less to an agent. Clearly the presence of information decay affects to whom an agent wants to link.

In particular, information decay has two effects. *First*, ex-ante homogeneous players become heterogeneous by their position in the network. Consider Fig. 1. Suppose Frank wants to connect to John, Susan and Pete. Without decay he does not care to which of these persons he creates a link. Any one of these links would give him the full benefits of being connected to all three. With decay, Frank strictly prefers a link with Susan, as she is in the middle. A link with either John or Pete would result in a higher loss due to decay. Thus a player with many direct links in the network, or a player who is in the middle, may be more attractive to sponsor a link to. *Second*, decay may give the individual player an incentive to sponsor links to players he is already connected to, but indirectly. Consider John in Fig. 1. John may prefer to create a link with Pete even though John and Pete are already connected via Susan. The reason is that the costs of the additional link may be worthwhile because it reduces the information loss due to decay. This second effect only affects the set of equilibrium networks if there is enough decay. In contrast, the first effect exists for all positive levels of decay. Small decay refers to those levels of decay for which the second effect does not affect the results. It follows that it is possible to analyze the first effect of decay in isolation. This is what we do in this paper.

We study the effects of small decay on network architecture in the seminal two-way flow model of Bala and Goyal (2000a). In the two-way flow model agents incur a private cost to link to other agents. The other agent always accepts the link.[5] Once linked, information flows in both directions. We characterize the network architectures which can be supported by (strict) Nash equilibria, and refer to such networks as (strict) Nash networks. We also provide a sufficiency result for general payoff functions. We find that (strict) Nash networks can have high diameters[6] if decay is small. In contrast, earlier literature on decay in the two-way flow model only provides characterizations and examples of strict Nash networks (henceforth SNNs, SNN for singular) with low diameters.

For this reason, we analyze whether high diameter networks can be stochastically stable when we introduce a simple myopic best reply dynamic. Moreover, we study how the returns to information affect which of these networks are SNNs for the widest range of parameters. In both these analyses, we restrict ourselves to benefit functions which are either strictly concave, linear or strictly convex. We show that networks of any diameter can be stochastically stable. Moreover we show that (i) the periphery-sponsored star[7] (PSS) is stable for the widest range of parameters, and (ii) that low diameter networks are relatively more likely under convex benefit functions, while (iii) high diameter networks are relatively more likely if the benefit function is concave.

Our paper is most closely related to Bala and Goyal (2000a) (henceforth BG), Hojman and Szeidl (2008) and Feri (2007). BG introduce the two-way flow model we use. They show that without decay, all non-empty Nash networks are minimally

[4] In the US there were roughly 270,000 conferences with in total about 61 million visitors during 2013, according to a report by PwC (*The economic significance of meetings to the U.S. economy: Interim study update for 2012*). Including other meetings at contracted venues such as business meetings and trade shows, there are 1.8 million events and about 225 million visitors.

[5] In the two-way flow model, only the player who initiates the link (the sponsor) pays the costs of the link. The other player, the link's recipient, incurs no costs for the link. Thus no recipient of a link has a reason to refuse that link. More broadly, given that information flows in two directions, the recipient of a link will always want to accept a link if the costs of accepting a link are sufficiently low. Thus this model captures situations in which the sponsor of a link bears the brunt of the linking costs.

[6] The diameter of a network is the largest distance between any two players in the network. The distance between two connected players is the shortest path of links in the network that connects the two players. See also Section 2.

[7] A periphery-sponsored star is a network in which there is a unique player to whom every other player sponsors a link, and in which no other links are sponsored.