



# Imitation and efficient contagion<sup>☆</sup>



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## ABSTRACT

This paper is about the diffusion of cooperation in an infinite population of networked individuals repeatedly playing a Prisoner's Dilemma. We formulate conditions on payoffs and network structure such that, starting from an initial seed group, imitative learning results in the overall adoption of cooperation—efficient contagion. Key to this result is the pattern of interaction among players who are at the same distance from the initial seed group. We find that the more these agents interact among themselves rather than with players who are closer to or further away from the initial seed group, the easier it is for efficient contagion to take place. We highlight the importance of cycles for efficient contagion, and show that the presence of critical edges prevents it. We also find that networks organized as dense clusters sparsely connected to one another tend to resist efficient contagion. Finally, we find that the likelihood of efficient contagion in a network increases when information neighborhoods extend beyond interaction neighborhoods.

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## 1. Introduction

Most economic and social interactions take place locally, within restricted subsets of a larger population. Interactions occur among relatives, friends, colleagues, business partners, neighbors in geographic or cultural space, etc. The set of pairwise relationships between agents constitutes a network, within which knowledge, information and other resources can be shared, exchanged and produced, new behaviors can be learned and tested, and economic value in general is created.<sup>1</sup>

When it comes to the diffusion of new behavior or technology, the existence of a fixed local interaction structure implies that diffusion takes place gradually. Starting from an initial group of individuals, diffusion progresses through the population like an epidemic, as contacts occur between infected and susceptible neighbors at increasing distance from the first infected individuals. There is however a considerable difference with the spread of epidemics. When exposure is all that is required, the speed and extent of contagion increase when edges are as “global” as possible, connecting players randomly across the network with as few local cycles as possible. By contrast, when the decision to adopt is based on payoff comparisons, the extent and speed of contagion depend on much finer structural properties of the iterated neighborhoods of the initial seed group.

The focus of the paper is on the Prisoner's Dilemma. In the setting considered, in any period, all agents play simultaneously the same, one-shot, two-by-two symmetric game with each of their neighbors in the network. In addition to playing, agents

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<sup>1</sup> Different neighborhoods can serve different purposes: typically, information could be gathered from a wider neighborhood than the one over which interaction takes place. We come back to this in [Section 3.2.2](#).

are learning. This they do through imitation, adopting the strategy of their best neighbor. Oftentimes in the literature agents imitate the average-payoff-maximizing strategy—instead, we simply assume that agents imitate the strategy with largest payoff, a behavior which research in psychology has found to be common. We formulate results on imitative contagion by efficient behavior in arbitrary regular networks.<sup>2</sup> These results exploit the structure of the iterated neighborhoods of a finite seed group of efficient players. The intuition is as follows. Cooperation generates higher payoffs than defection if players are segregated, i.e., when players interact mostly with players of their kind at any stage of contagion (and thus the cooperators are relatively well protected from exploitation by defectors). This takes the form of a constraint on the number of interactions with players at the same distance from the initial seed group. We show that this constraint more generally relates to the presence of cycles in the network. Trees, i.e., networks without cycles, do not permit efficient contagion. Likewise, the existence of critical edges prevents contagion—so not only should there be cycles, but each agent should be part of at least one of them. We also find that a set of individuals which is only sparsely connected to the rest of the network will resist efficient contagion. This could be bad for social networks, as they often display the characteristic features of a small world—sparse networks consisting of dense clusters of individuals connected by a few clique-spanning ties. Regarding the likelihood of contagion in a given network, we contend that longer cycles are more sensitive to the choice of the initial seed group than shorter ones. Therefore, even if short cycles are by no means necessary, it is likely that they are more robust to the choice of the seed group. As a consequence, although clustering (neighbor commonality) is also not necessary, it can support contagion. Lastly, distinguishing the interaction and information neighborhoods in the vein of recent research (see the discussion below), we find that iterated neighborhoods should grow fast for efficient behavior to spread, a situation which is perfectly compatible with the absence of cycles.

This paper is obviously related to a large set of references on imitation and local interaction in two-by-two symmetric games. We only review this literature very selectively, emphasizing differences from or similarities with our findings as these are discussed. The next section presents the model and introduces the notion of shell-wise contagion. We then move on to the results, discussing first the condition on iterated neighborhoods, and then examining its more general network implications, before finally distinguishing the interaction and information neighborhoods.

## 2. The model

### 2.1. Network definitions

We consider a countably infinite set  $S$  of nodes (players) and a set  $g$  of edges (links) between unordered pairs in  $S$ . The pair  $(S, g)$  defines a *network*. Letting  $ij$  denote the link between  $i$  and  $j$ ,  $ij \in g$  indicates that  $i$  and  $j$  are linked in the network  $(S, g)$ . The *neighborhood* of  $i$  consists of the players to whom  $i$  is directly linked, and is denoted  $N_i = \{j \neq i : ij \in g\}$ . The size of the neighborhood of  $i$  is the number of links held by  $i$ , also called its *degree*,  $n_i = \#N_i$ . Our focus is on regular networks, i.e., networks in which degree is identical across nodes:  $n_i = n, \forall i \in S$ . The neighborhood of a set of players is the union of neighborhoods of the players in that set.

A *path* is a sequence of links  $i_1i_2, \dots, i_{p-1}i_p$  between unordered pairs in  $S$  such that  $i_l i_{l+1} \in g$  and  $i_l \neq i_{l+1}$  for all  $l = 1, \dots, p$ . The length of a path is the number of edges it contains. A *cycle* of length  $p$  is a path of length  $p$  such that the first and last nodes coincide:  $i_1 = i_p$ .

The number of links in the shortest path between  $i$  and  $j$ , denoted  $d_{ij}$ , is the (geodesic) *distance* between  $i$  and  $j$  in the network  $(S, g)$ . We have  $d_{ij} = 1, \forall j \in N_i$ . We suppose that  $d_{ij} < \infty, \forall i, j \in S$ , i.e., the network is *connected*.

Consider now an arbitrary finite group  $X \subset S$  of players. The set  $X$  will play the role of the seed group, i.e., the set of players starting the epidemic. The distance between  $X$  and player  $i \notin X$  is  $d_{X,i} = \min_{j \in X} \{d_{ij}\}$ . We write  $N_X^d$  for the  $d$ -iterated neighborhood

of  $X$ ,  $N_X^d = \{i \in S : d_{X,i} = d\}$ , with  $d \geq 0$  and  $N_X^0 = X$ . We define the  $d$ -*shell* of  $X$  as the set of players who are *within* distance  $d$  of  $X$ , augmented with  $X$ , as

$$X_d = \bigcup_{0 \leq l \leq d} N_X^l. \tag{1}$$

The  $d$ -*shell* of  $X$  is the union of  $X$  and its iterated neighborhoods up to distance  $d$ .

We finally introduce a notation specific to the problem we consider. Let

$$N_{i,X}^k = \{j \in N_i : d_{X,j} = d_{X,i} + k\}, \tag{2}$$

with  $k \in \{-1, 0, +1\}$ , be the neighbors of  $i$  whose distance to  $X$  equals  $i$ 's distance to  $X$  augmented with  $k$ . We can now define, for a given finite set  $X$  and any  $i \in S$ , the *in-degree* of  $i$  as  $n_{i,X}^{-1} = \#N_{i,X}^{-1}$ , i.e., the number of neighbors of  $i$  who belong to the previous iterated neighborhood of  $X$ . Similarly we define the *out-degree* of  $i$  as  $n_{i,X}^{+1} = \#N_{i,X}^{+1}$ , and the *intra-degree* of  $i$  as  $n_{i,X}^0 = n - n_{i,X}^{-1} - n_{i,X}^{+1} = \#N_{i,X}^0$ . In-, and out- and intra-degree are all with respect to  $X$ , which this notation makes clear; they play a particularly important role in the discussion of contagion below.

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