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Level-*k* analysis of experimental centipede games[☆]

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ABSTRACT

As one of the best-known examples of the paradox of backward induction, centipede games have prompted a host of studies with various approaches and explanations (McKelvey and Palfrey, 1992; Fey et al., 1996; Nagel and Tang, 1998; Rapoport et al., 2003; Palacios-Huerta and Volij, 2009). Focusing on initial plays observed in experiments, this paper attempts to offer another explanation based on thorough study of level-*k* models as applied to these games. Borrowing ideas from the cognitive hierarchy model (Camerer et al., 2004), the authors constructed a group of models based on levels of rationality, and also tested for various assumptions on the play of the most naïve player type in these models. It was found that level-*k* models generally perform better than the agent quantal response equilibrium (AQRE) model and its variant with altruistic player types for increasing-pie centipede games, while the AQRE model with altruistic player types performs better in constant-pie games.

1. Introduction

The centipede game, first introduced by Rosenthal (1982), is one of the most celebrated examples of the paradox of backward induction. In its two-player version, the game proceeds as follows: Beginning with player 1, the two players alternately choose to either “Take” (hereafter T) or “Pass” (hereafter P) a finite number of times; as soon as either player chooses T, the game ends. This game has a characteristic payoff structure: The last player is better off choosing T at the last node; given that all players choose T at later nodes, selecting T is also better for the player whose turn it is to move at the node. Thus, by backward induction, every player choosing T at every node is the unique subgame perfect equilibrium (SPE), leading to the outcome that the game ends immediately after player 1 chooses T at the first node. While this line of rational reasoning sounds quite compelling, it seems to be somewhat at odds with our intuition. The rational prediction sounds counterintuitive, especially when the “pie” (the sum of all players’ payoffs) to be shared grows with each node. For this reason, centipede games have attracted the attention of numerous game theorists and experimental economists.

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Since the publication of the seminal paper by McKelvey and Palfrey (1992), several experiments have been conducted in various settings with focus on factors such as the number of moves in the game, how the pie changes as the game proceeds, the number of players, and the number of repeated experimental rounds (McKelvey and Palfrey, 1992; Fey et al., 1996; Rapoport et al., 2003). As explained later in more detail, in spite of these diverse settings, the experimental results mostly reveal considerable deviations from SPE prediction; a non-negligible proportion of players usually continues to choose P until the middle nodes of the game are reached. Each of the above-cited papers offers its own theoretical explanation for this phenomenon. While Rapoport et al. (2003) focused on the learning dynamics of observed plays[2], McKelvey and Palfrey (1998) and Fey et al. (1996) proposed a static equilibrium concept known as the agent quantal response equilibrium (AQRE) model to explain their data[3]. McKelvey and Palfrey (1992, 1998) also introduced a fraction of altruistic player type[4] to their AQRE model (referred to here as AQRE+) and found that it significantly increased goodness-of-fit for the data[5].

The present paper explores the possibility of an alternative explanation based on level-k theory for experimental centipede games. The level-k model assumes that each subject is one of several types of player with differing degrees of rationality. That is, it first specifies play of the most naïve type (level 0), and sets the level-k type's play as the best response to the play of the level-($k-1$) type, where $k \geq 1$. Based on this theory, we can use observed data to statistically estimate the distribution of types among subjects. Thus, it is a non-equilibrium theory that assumes players are self-interested, but may be boundedly rational in the sense that their beliefs are out of equilibrium. Accordingly, the concept forms part of the recent approach of attempting to explain observed plays in terms of boundedly rational beliefs rather than in terms of other-regarding preferences such as fairness or altruism on the part of players.

In order to thoroughly examine the explanatory power of level-k theory, we constructed a group of level-k models, some of which incorporated ideas from the cognitive hierarchy model, and applied these models to the results of our own experimentation as well as to those of previously published works. We also tested for various assumptions on the play of the most naïve player type in these models. Thus we systematically examined the performance of the models in explaining first-round data from various experiments, and compared their performance to those of the AQRE and AQRE+ models.

The data examined from past studies included those of the six-move game experiment described by McKelvey and Palfrey (1992), those of Palacios-Huerta and Volij (2009) (except those in the field experiment with chess masters), and those of Levit et al. (2011). The chess-master data from Palacios-Huerta and Volij (2009) were excluded because of their apparent exceptionality in that the observed plays were very close to those predicted by SPE, while the data obtained in the replication study by Levit et al. (2011) show no such tendency.

We found that level-k models do a better job of predicting the observed outcomes than the AQRE+ model and the original AQRE model for the class of increasing-pie centipede games. Thus, the seemingly fairness-oriented outcomes observed in increasing-pie games can be well explained using level-k models. We also found, however, that the AQRE+ model outperforms a group of level-k models for constant-pie games, where observed plays are close to those predicted by SPE.

This paper is organized as follows: Section 2 outlines the models to be compared in terms of econometric estimation; a group of level-k models, the AQRE model and the AQRE+ model. Section 3 reports on our own experiments and compares the performance of the above models in explaining our data. Section 4 presents econometric comparisons of the same models using past experimental results. Some discussions on the relation of our study to the past literature are provided in Section 5, which is followed by the Conclusion.

2. Models

2.1. A group of level-k-related models

The level-k model assumes that each subject's decision rules follow one of a small set of *a priori* plausible behavior types, and tries to estimate the distribution of player types that best fits the observed data. In the basic level-k model, the play of each player type is defined inductively; type $Lk(k \geq 1)$ of player i plays the best response to the play of type $L(k-1)$ of player j ($j \neq i$), where type $L0$'s action for each player is predetermined in the inductive process. Thus, given the specification of

[2] For the reason to be explained later, our focus is on initial plays in centipede games. However, it should be noted that we do not seek to downplay the importance of learning in general. The reasons for our focus on initial plays are as follows. While the learning models proposed so far give a good picture of players' learning processes in experimental centipede games, such processes are known to be strongly dependent on initial plays and beliefs, as highlighted by Roth and Erev (1995). Systematic deviations from the equilibrium prediction observed in the earlier rounds should certainly affect the course of the learning process. We therefore believe that our analysis helps to shed light on the learning process despite not dealing directly with learning itself.

[3] The model proposed by Zauner (1999) may also be included in this strand, as it is based on equilibrium with payoff perturbations. In our initial computation for a four-move game, the goodness of fit of this model was no better than that of AQRE. Accordingly, it was excluded from our consideration.

[4] To the best of our knowledge, other models proposed to explain observed plays in centipede games assume that players have other-regarding preferences. In this strand of research, Dufwenberg and Kirchsteiger (2004) apply sequential reciprocity equilibrium to a centipede game with linearly increasing payoffs. They show that there is a unique equilibrium with which both players stay in the game until the last node if at least one of them is sufficiently motivated by reciprocity. However, this characteristic seems to be at odds with the commonly observed property by which the conditional probability of T increases in the later nodes of increasing-pie centipede games. Furthermore, since the parameters of their model are unobservable and cannot be identified from the data, it is not amenable to econometric evaluation. Accordingly, we did not consider their model for econometric comparison.

[5] It should be noted, however, that introducing altruistic behavior into QRE models is not without criticism from an econometric point of view (Haile et al., 2008).