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Research report

Weighted Schatten p-norm minimization for 3D magnetic resonance images denoising

Lin Zhai[a], Shujun Fu[a,*], Hongli Lv[a], Caiming Zhang[b], Fengling Wang[c]

[a] School of Mathematics, Shandong University, Jinan 250100, China
[b] School of Computer Science and Technology, Shandong University, Jinan 250101, China
[c] College of Arts Management, Shandong University of Arts, Jinan 250300, China

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ABSTRACT

Magnetic resonance (MR) imaging plays an important role in clinical diagnosis and scientific research. A clean MR image can better provide patient's information to doctors or researchers for further treatment. However, in real life, MR images are inevitably corrupted by annoying Rician noise in the process of imaging. Aiming at the Rician noise of 3D MR images, a framework is proposed to suppress noise by low-rank matrix approximation (LRMA) with weighted Schatten p-norm minimization regularization (WSNMD-3D). The proposed method not only considers the importance of different rank components, but can also approximate the true rank of the latent low-rank matrix. This approach first groups similar non-local cubic patches extracted from the noisy 3D MR image into a matrix whose columns are vectorized patches. The above matrix can be modeled as a low-rank matrix approximate model. Then weighted Schatten p-norm minimization (WSNM) is applied to the model, which shrinks different rank components with different treatments. Finally, the denoised 3D MR image is acquired by aggregating all denoised patches with weighted averaging. Experimental results on synthetic and real 3D MR data show that the proposed method obtains better results than state-of-the-art methods, both visually and quantitatively.

1. Introduction

Magnetic resonance (MR) imaging is a medical imaging technique used in radiology to visualize detailed internal structure of the body (Arab et al., 2018). The MR image has been widely used for clinical diagnosis and scientific research. However, due to the imaging characteristics of MR images, noise will always be introduced in the process of image acquisition, especially under the requirements of high speed capture or high resolution. Accuracy of clinical diagnosis and the effectiveness of automated computer analysis are reduced due to the adverse effects of noise on image quality, contrast, and analytical structure. So noise reduction plays an important role in MR image analysis. Noise model in MR image is assumed as Rician distribution (Nowak and Wavelet-based, 1999) which is different from the natural images whose noise is often modeled as Gaussian distribution. Therefore, removing the Rician noise in MR images is still a challenging topic.

Generally speaking, the denoising methods for MR images are mainly divided into two types (Mohan et al., 2014): One way is to acquire the objective several times and average them. However, this will increase the acquisition time and the objective must remain stationary in the whole acquisition process. Another way is to denoise the MR images by using the post processing algorithms which will not effect the acquisition time.

Nowadays, a lot of excellent algorithms are proposed for MR image denoising, such as gradient-based methods, statistic-base methods and transform domain-based methods. For gradient-based methods, filters based on anisotropic diffusion (AD) and total variation (TV) regularization can reduce noise and retain important image structures as well (Gerig et al., 1992; Sijbers et al., 1999; Liu et al., 2014b). However, gradient-based methods always erase small image features due to blocky (staircase) effect cased by edge enhancement. Statistic based approaches are also widely used in MR image denoising area because of their good performance for removing Rician noise (Awate and Whitaker, 2007; Rajan et al., 2012; Aja-Fernandez et al., 2008). But owing to the underlying assumption of neighborhood stationarity, small details are often lost while removing the Rician noise. In addition, wavelet and other transform-based methods, such as contourlet and curvelet, play an important role in MR image denoising field (Pizurica et al., 2003; Anand and Sahambi, 2010). Although fixed bases of the above transform-based methods bring excellent denoising performance,

* Corresponding author.
E-mail address: shujunfu@163.com (S. Fu).

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visual artifacts will be introduced due to using a fixed basis to represent local structures presented in MR images. To overcome the drawback, many data-driven based methods are proposed, such as singular value decomposition (SVD) (Zhang et al., 2015), principal component analysis (PCA) (Manjón et al., 2013) and dictionary learning (DL) (Wang et al., 2013). These methods share a similar strategy that shrinks the noise-dominant components by thresholding algorithm to approximate noise-free components in transform domain. Except for the methods above, non-local mean (NLM) based methods considering the non-local similarity and redundancy of the image have provided excellent performance in MR image denoising (Manjón et al., 2008). Recently, methods based on non-local similarity and signal sparseness have obtained remarkable achievements, such as 3D non-local transform-domain filter (BM4D) (Maggioni et al., 2013) and PRI non-local principal component analysis (PRI-NL-PCA) (Manjón et al., 2015).

More recently, low-rank matrix approximate (LRMA) that aims to recover the latent low-rank matrix structure from its noisy observation, has attracted increasing attentions in image processing due to its popularity and effectiveness. In general, LRMA can be implemented by low-rank matrix factorization approaches (Buchanan and Fitzgibbon, 2005; Ke and Kanade, 2005) as well as rank minimization approaches, the latter are our main research object. Because it is a NP hard problem of direct rank minimization, the problem is usually relaxed by substitutively minimizing the nuclear norm of the estimation matrix. This solution is a convex relaxation of minimizing the matrix rank and is known as nuclear norm minimization (NNM). The nuclear norm is defined as the sum of the singular values of a matrix $X \in \mathbb{R}^{m\times n}$, *i. e.*, $||X||_* = \Sigma_i \sigma_i(X)$, where $\sigma_i(X)$ represents the i-th singular value of X and $i = 1, \ldots, r$, $r = \min\{n, m\}$. Given a matrix Y, the goal of NNM is to find a low-rank matrix X that satisfies the following objective function:

$$\hat{X} = \underset{X}{\operatorname{argmin}} \|X - Y\|_F^2 + \tau \|X\|_*, \tag{1}$$

where τ is a regularization parameter to balance the data fidelity and regularization, and $\|\cdot\|_F$ is *Frobenius* norm. Many models based on NNM have been proposed in recent years (Ji and Ye, 2009; Lin et al., 2009; Cai et al., 2008). For example, (Cai et al., 2008) proved that the NNM can be easily solved by a soft-thresholding operation.

Despite the convexity of the NNM model, the recovery performance of the convex relaxation may be reduced in the measurement of noise, and the solution may seriously deviate from the original solution of rank minimization problem (Liu et al., 2014a; Nie et al., 2012; Mohan and Fazel, 2012). Therefore, a restricted Schatten p-norm minimization is proposed by (Liu et al., 2014a; Nie et al., 2012; Mohan and Fazel, 2012) to solve the problem of non-convex relaxation of the rank minimization. The Schatten p-norm minimization provides better approximation to the original NP hard problem, and obtains better theoretical and practical results than the standard NNM (Xie et al., 2015). The Schatten p-norm with $0 < p \le 1$ is defined as:

$$\|X\|_{\mathrm{Sp}} = \left(\sum_{i=1}^{r} \sigma_i^p\right)^{\frac{1}{p}}. \tag{2}$$

In theory, Schatten p-norm will ensure a more accurate recovery of the signal, while requiring only a *weaker restricted isometry property* than the traditional trace form (Liu et al., 2014a). Xie et al. (2015) and Lu et al. (2015) had proved that the Schatten p-norm based model exceeds the standard NNM.

Although NNM and Schatten p-norm minimization cause widespread concern in academia due to their validity and rigorous theoretical derivation, they still have some limitations. That is, traditional NNM and Schatten p-norm minimization treat all singular values equally and shrink them with the same thresholding. But actually, the large singular values of a data matrix provide the significant edge and texture information. That means when the image is denoised from its observation, larger singular values should be shrunk less, while small singular values are just the opposite. Traditional NNM and Schatten p-norm minimization models are not flexible enough to deal with this problem. To overcome the drawback of NNM, weighted nuclear norm minimization (WNNM) is proposed by Gu et al. (2017). Weighted nuclear norm is defined as

$$\|X\|_{\omega,*} = \sum_{i=1}^{r} \omega_i \sigma_i(X), \tag{3}$$

where $\omega = [\omega_1, \ldots, \omega_r]^{\mathrm{T}}$ is a non-negative vector and $\omega_i \ge 0$ is the weight assigned to $\sigma_i(X)$. Compared with NNM, WNNM assigns different weights to different singular values and can better protect image details while removing noise. Weighted nuclear norm proximal (WNNP) operator is the key step of solving general WNNM model, which determines the general solving regime of the WNNM problem:

$$\hat{X} = \underset{X}{\operatorname{argmin}} \|X - Y\|_F^2 + \|X\|_{\omega,*}. \tag{4}$$

Although non-convex, it is proved that WNNP is equivalent to a standard quadratic programming problem with linear constraints, which helps to solve the original problem by using the existing convex optimization solvers (Gu et al., 2017). It is worth mentioning that when the weights are sorted in a non-descending order, the optimal solution can be easily obtained in closed-form.

Inspired by WNNM and Schatten p-norm minimization, weighted Schatten p-norm minimization (WSNM) (Xie et al., 2015) is proposed for LRMA. It is proved that WSNM is more flexible in handling different rank components and provides better approximation to the original LRMA problem than WNNM and other NNM-based methods. In the solution phase, under certain weights permutation, WSNM can be equivalently converted into independent non-convex l_p-norm subproblems, whose global optimum can be effectively solved by generalized soft-thresholding algorithm (GST) (Zuo et al., 2013).

At present, most Rician noise denoising algorithms have been vertified that they can remove the noise efficiently. However, they are not as good in terms of details and structural protection due to lacking of the ability to capture detailed image information. To overcome the above disadvantages, we propose a LRMA model using WSNM regularization for Rician noise reduction in 3D MR images (WSNMD-3D). When the matrices are formed by similar noisy cubic patches extracted from the noisy image, the noise reduction problem can be modeled as a LRMA model. WSNM is applied to the model, shrinks different rank components with different treatments. The final denoised 3D MR image is acquired by aggregating all denoised patches with weighted averaging. Experiments on synthetic and real 3D MR data indicate that the proposed method outperforms other state-of-the-art methods.

The structure of this paper is as follows: In Section 2, we briefly introduce the WSNM model and its solution, WSNM-based denoising algorithm on 3D image (WSNMD-3D), and the application of WSNMD-3D on 3D MR image. In Section 3, we show experimental results and discuss the merits of the proposed approach with respect to other methods. Finally, we conclude our paper in Section 4.

2. Material and methods

2.1. Weighted Schatten p-norm minimization

2.1.1. LRMA-based denoising

Grouped similar patches always share similar underlying image structures. Therefore, LRMA can be used to recover true noise-free image patches by low-rank modeling of non-local similarities. Usually, matrix formed by vectorized similar patches can be considered as a low-rank approximation of a noisy version.

Supposing we have an observation noisy image patch matrix $Y \in \mathbb{R}^{m\times n}$, the following model can be used to approximate the original noise-free image information by finding out the latent low-rank matrix (Xia et al., 2017):