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On the complicated price dynamics of a simple one-dimensional discontinuous financial market model with heterogeneous interacting traders☆

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ABSTRACT

We develop a financial market model with heterogeneous interacting agents: market makers adjust prices with respect to excess demand, chartists believe in the persistence of bull and bear markets and fundamentalists bet on mean reversion. Moreover, speculators trade asymmetrically in over- and undervalued markets and while some of them determine the size of their orders via linear trading rules others always trade the same amount of assets. The dynamics of our model is driven by a one-dimensional discontinuous map. Despite the simplicity of our model, analytical, graphical and numerical analysis reveals a surprisingly rich set of interesting dynamical behaviors.

1. Introduction

Spectacular financial market bubbles have repeatedly been observed in the past, often followed by equally stunning crashes. In some cases, these events even had an impact on the real economy, triggering deeper recessions, for instance. Moreover, the volatility in financial markets may be regarded as excessively high in the sense that prices fluctuate more strongly than warranted by the underlying fundamentals. Also extreme price changes, which make up a large part of financial market risk, occur quite frequently. Detailed empirical accounts on these intriguing phenomena are provided by Sornette (2003); Shiller (2005, 2008) and Lux (2010).

Obviously, it is important to understand what drives the dynamics of financial markets. Bouchaud et al. (2009) present significant empirical evidence showing that asset prices mainly adjust with respect to the markets' order imbalances which,

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of course, originate from the transactions of its market participants. Fortunately, we at least have some empirical evidence on how agents determine their speculative orders. As can be seen from empirical studies involving questionnaires (summarized by Menkhoff and Taylor, 2007), market participants rely on both technical and fundamental trading rules to determine the course of the market. Technical analysis is a trading method that seeks to identify trading signals from past price movements (Murphy, 1999). As a result, technicians - also called chartists - may have a destabilizing effect on the dynamics of financial markets. Fundamental analysis presumes that prices will mean-revert toward fundamental values (Graham and Dodd, 1951), generally inducing some kind of market stability. Similar insights are obtained in laboratory experiments in which human subjects trade in a controlled financial market environment (Smith et al., 1988; Hommes et al., 2005).

But how exactly do markets with a diverse ecology of interacting technical and fundamental traders function? Models with heterogeneous agents take exactly this issue into account. For recent surveys of this burgeoning field of research, see Chiarella et al. (2009); Hommes and Wagener (2009); Lux (2010) and Westerhoff (2009), among others. While some stochastic versions of these models aim at matching the stylized facts of financial markets – several interesting contributions are presented in LeBaron (2006); Lux (2009) and Chen et al. (2010)[1] – other studies focus on deterministic setups to improve our basic knowledge of what drives prices in financial markets.

Let us briefly outline a few of these frameworks in order to appreciate the insights made in this exciting field of research and to clarify the extent to which our model differs from previous works in this field.

- One interesting finding is due to Day and Huang (1990), who show that endogenous price dynamics may be triggered by nonlinear trading rules. In their model, chartists apply a linear trading rule, and their orders destabilize the market close to the fundamental value. The trading behavior of fundamentalists is nonlinear. The more the price deviates from the fundamental value, the more aggressive they become. Eventually, orders placed by fundamentalists exceed orders placed by chartists, and prices are pushed back towards fundamental values. However, close to the fundamental value, chartists again dominate the market and the process repeats itself, albeit in an intricate, unpredictable way. Related models featuring nonlinear technical trading rules have been elaborated by Chiarella (1992); Chiarella et al. (2002), and others.
- Another interesting insight is that when agents switch between technical and fundamental analysis, a similar dynamic behavior can emerge. Let us suppose the market is dominated by destabilizing chartists. In this case, it is likely that prices disconnect from fundamentals. However, when fundamental analysis becomes more popular, a period of price stability, together with a convergence towards fundamental values, may set in. Brock and Hommes (1998) develop a model in which agents switch between trading rules with respect to their past performance and thus display some kind of learning behavior. In Kirman (1991), agents have social interactions that may lead to swings of opinion. In Lux (1998), traders compare the performance of trading rules but are also subject to herding behavior.
- A third natural mechanism of endogenous dynamics is based on market interactions. Let us assume a situation in which technical traders can switch between several financial markets. A market may temporarily become unstable if it attracts numerous chartists from other markets. However, when chartists leave the market again – e.g. when other markets appear to be more profitable – a period of convergence sets in. Models along these lines have been proposed by Westerhoff (2004); Chiarella et al. (2005) and Westerhoff and Dieci (2006).

The contribution our paper makes is as follows: we develop a novel financial market model with five different types of agents. Technical traders believe in the persistence of bull and bear markets. For instance, these traders optimistically buy assets in a bull market. In contrast, fundamental traders expect prices to return towards fundamental values. In a situation where the market is overvalued (i.e. in a bull market), fundamentalists submit selling orders. Although these two building blocks are standard in the literature, we generalize them in our paper. First, speculators react asymmetrically in bull and bear markets. Here is an example: fundamentalists may trade more (less) aggressively when an asset is overvalued by 10 percent than when it is undervalued by 10 percent.[2] Second, some speculators determine the size of their orders using linear trading rules. However, other speculators simply keep the size of their orders constant (always trading the same amount of assets) and only determine the direction of trade with their pertinent trading philosophy. Hence, there are two types of technical and two types of fundamental traders. Finally, a market maker, the fifth type of agent, adjusts prices with respect to excess demand in the usual way.

Interestingly, our simple setup constitutes a one-dimensional discontinuous dynamical system which is sufficient to generate a very rich set of dynamical phenomena, including, for instance, irregular fluctuations between bull and bear market regimes, as observed in real markets and first modeled by Day and Huang (1990), yet in a different model environment. This does not, however, imply that the established and sophisticated mechanisms mentioned above do not play an important role in explaining the dynamics of financial markets. It does, however, demonstrate that at least part of the dynamics of financial markets may be due to rather simple deterministic mechanisms. In addition, our paper shows the relevance of discontinuous maps to the analysis of financial market dynamics, a rather new field of *applied* mathematics that has not yet yielded many results. Nevertheless, note that there are already several interesting economic models that feature piecewise-smooth or discontinuous maps, for example, the pioneering works by Day (1982, 1994); Day and Shafer (1987); Day and Pianigiani

[1] For specific examples see Gaunersdorfer and Hommes (2007); He and Li (2008); Franke and Westerhoff (2009) and Franke (2010).
[2] Such a modelling device is also used in Zhu et al. (2009).