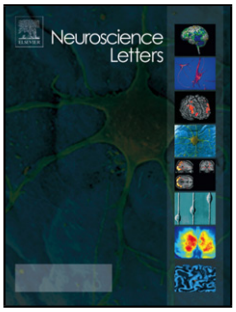

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EEG DYNAMICS ON HYPERBOLIC MANIFOLDS

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Highlights

- EEG dynamics take place in a three-dimensional Euclidean environment.
- They can tranferred to a hyperbolic, concave maniford with negative curvature.
- This transfer allows a better assessment and quantification of EEG traces' undetectable features.

Biological activities, including cellular metabolic pathways, protein folding and brain function, can be described in terms of curved trajectories in hyperbolic spaces which are constrained by energetic requirements. Here, starting from theorems recently-developed by a deceased Field Medal young mathematician, we show how it is feasible to find and quantify the shortest, energy-sparing functional trajectories taking place in nervous systems' concave phase spaces extracted from real EEG traces. This allows neuroscientists to focus their studies on the few, most prominent functional EEG's paths and loops able to explain, elucidate and experimentally assess the rather elusive mental activity.

KEYWORDS: topology; Mirzakhani; brain; hyperbolic; manifold; polynomials

The scientific knowledge of physical dynamics is currently more advanced than their biological counterparts, the latter still lacking the required explanatory depth. One of the main reasons for this disparity is that, while physics has been provided with powerful mathematical tools, biology has not. Physics works much better in the description of natural features, because we have recognized its subtending manifolds. To make an example, both general relativity and quantum mechanics describe Nature using mathematical structures, such as tensors and probability theory (Yilmaz, 1982; Comte, 1996; Fre, 2013). Their subtending manifolds (*i.e.*, the phase spaces in which the corresponding activities take place) are well-known and have been experimentally confirmed: general relativity is described on a 3+1 dimensional pseudo-Riemannian manifold with tensor fields obeying certain partial differential equations (see, e.g., Weyl, 1955; Weyl, 1988), while quantum field theory is portrayed on an R^4 projective Hilbert space, with operator-valued fields obeying certain Lorentz-invariant partial differential equations and commutation relationships (Tegmark 2008).

Concerning biology, to make an example, it is still difficult to understand the information content endowed in brain electroencephalogram (EEG) traces, or to find a definition for life (McKay, 2004; Trifonov, 2012). There is no unequivocal definition of such biological dynamics, despite many proposals have been suggested. Apart from the intrinsic problems in describing such elusive phenomena, the very current limit is that the most of the proposed features - from the concept of homeostasis to organization, from adaptation to response to stimuli - are just descriptive. In sum, in neuroscience, as in biology in general, we lack the proper manifolds, and therefore we lack mathematically precise objectivity. This is a huge restraint, because the sole language able to describe in quantitative terms scientific issues is the mathematical one. If we leave apart math, we do not have the proper observables, and neural dynamics are made by observables.

The aim of this paper is to make an effort to provide a mathematical, operational, quantifiable definition of the dynamics of EEG real traces. In particular, to assess neurophysiological issues, we need at first to find the proper manifolds where their operations might take place. Therefore, our goal is to describe these phase spaces, by treating brain EEG activities and their corresponding gradient-descent Langevin equation, in terms of algebraic topology.