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On the stability of Cournot equilibrium when the number of competitors increases

Tönu Puu *

CERUM, Umea University, SE-90187 Umeå, Sweden

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Abstract

This article reconsiders whether the Cournot equilibrium really becomes a perfect competition equilibrium when the number of competitors goes to infinity. It has been questioned whether the equilibrium remains stable with an increasing number of firms. Contraindications were given for linear and for isoelastic demand functions. However, marginal costs were then taken as constant, which means adding more potentially infinite-sized firms. As we want to compare cases with few large firms to cases with many small firms, the model is tuned so as to incorporate capacity limits, decreasing with an increasing number of firms. Then destabilization is avoided.

Keywords: Cournot oligopoly; Capacity limits; Stability

1. Introduction

An intriguing question in microeconomics has been whether an increase in the number of competitors in a market always defines a path from monopoly, over duopoly, oligopoly, and polypoly using Frisch's term (see Frisch, 1933), to perfect competition. There are two different issues involved: (i) the Cournot equilibrium (see Cournot, 1838) must have the competitive equilibrium as its limit, and (ii) the increasing number of competitors must not destabilize that equilibrium state. As a rule the first question is responded to in the affirmative, whereas there have been raised serious doubts about the second; examples are Palander (1939), Theocharis (1959), Agiza (1998), and Ahmed and Agiza (1998). Theocharis pointed out that an oligopoly system with n

* Tel.: +46 90 786 9819; fax: +46 90 786 5121.
E-mail address: tonu.puu@econ.umu.se.

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competitors, producing under constant marginal cost, and facing a linear demand function, would be only neutrally stable for three competitors and unstable for four and more competitors. This paradox has never been resolved, and is normally associated with the name of Theocharis.

The argument is very simple. With a linear demand function, the total revenue function for each competitor becomes quadratic in the supply of the firm itself and linear in the supplies of the competitors. Differentiating the profit function to obtain the marginal profit condition and solving for the firm's own supply as a function of the supplies of the competitors (for the reaction function) results in a linear function with the constant slope $-1/2$. Hence, the $n \times n$ Jacobian matrix, whose eigenvalues determine the stability of the Cournot point, has the constant $-1/2$ in all off-diagonal elements, and 0 in the diagonal elements.

Accordingly, the characteristic equation factorizes into:

$$\left(\lambda - \frac{1}{2}\right)^{n-1} \left(\lambda + (n-1)\frac{1}{2}\right) = 0,$$

so, the system has $n - 1$ eigenvalues $\lambda_1, \ldots (n-1) = 1/2$, and one eigenvalue $\lambda_n = -(n-1)(1/2)$. This last eigenvalue causes the trouble: for $n = 3$ we have $\lambda_3 = -1$, for $n = 4$ we have $\lambda_4 = -3/2 < -1$, and so forth.

Theocharis's argument was in fact proposed 20 years earlier by Palander (p. 237), who wrote: "as a condition for an equilibrium with a certain number of competitors to be stable to exogenous disturbances, one can stipulate that the derivative of the reaction function f' must be such that the condition $|(n-1) f'| < 1$ holds. If this criterion is applied to, for instance, the case with a linear demand function and constant marginal costs, the equilibria become unstable as soon as the number of competitors exceeds three. Not even in the case of three competitors will equilibrium be restored, rather there remains an endless oscillation".

It is difficult to say to what extent Palander's argument was widely known, but he had already presented a substantial part of it at a Cowles Commission conference in 1936.

Linear reaction functions (arising with linear demand functions) are easy to use in the argument because their slopes are constant, so one does not need to be concerned about the argument values in the Cournot equilibrium point. Linear functions are, of course, globally a problem, because, to get things proper, one has to state them as piecewise linear in order to avoid negative supplies and prices, but neither Palander nor Theocharis were concerned with anything but local stability.

However, the same properties were shown to hold by Agiza (1998), and Ahmed and Agiza (1998) for a non-linear (isoelastic) demand function and, again, constant marginal costs, suggested by the present author, Puu (1991). The model was originally suggested as a duopoly, Puu (1991), and then as a triopoly Puu (1996), and the focus was on the global complex dynamics and the bifurcations it gave rise to. Now the derivatives of the reaction functions are no longer constant but vary with the coordinates, and hence with the location of the Cournot point. However, it can easily be shown that if we assume the firms to be identical, then the problem of local stability becomes almost as simple as with linear demand functions. With n competitors, the derivative of the reaction functions in the Cournot point, the off-diagonal element in the Jacobian matrix, becomes $-((n-2)/(n-1))(1/2)$, and we get eigenvalues $\lambda_{1,\ldots(n-1)} = ((n-2)/(n-1)(1/2))$ and $\lambda_n = -(n-2)(1/2)$ so it is now the case $n = 4$ that is neutrally stable, whereas $n = 5$ and higher become unstable.

The assumption of identical firms also eliminates a problem that does not arise in the isoelastic case, but in many others, such as, the multiple intersection points of the reaction functions, studied