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Global stability of an information-related epidemic model with age-dependent latency and relapse☆

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ABSTRACT

We study the problem of information-related contact rate for an *SEIR* epidemic model with age-dependent latency and relapse. The contact pattern includes an information variable which is a negative feedback of susceptible individuals related to the memory of past and current states of infectious disease. We prove the asymptotic smoothness of solutions and the existence of equilibria in the positive invariant set. And we show uniform persistence of the system by comparing with a system of Volterra type integro-differential equations. Importantly, it follows from appropriate conditions that the local stability and global stability of equilibria can be concluded by the methods of corresponding characteristic equations and proper Lyapunov functionals, respectively. Finally, we give numerical simulations to illustrate the influence of the information on changing the coordinate of endemic equilibrium by increasing susceptible population and decreasing infectious population. It is remarkable to find that information coverage and the length of disease-relate memory would work effectively on the progression of disease.

1. Introduction

As the epochs of high prevalence risk of infectious diseases, such as influenza, SARS and tuberculosis (TB), mathematical models have emphasized an important and significant role played by the information of infectious spreading, as well as awareness campaign in restraining a massive contagion outbreak and using more safer prevention to reduce the risk of reappearance (Agaba et al., 2017b; Misra et al., 2011; Sahu and Dhar, 2015; Yan et al., 2016). It has been shown that people's response to the threat of disease rests with their perception of risk. The perceptive ability is influenced by public and private information disseminated widely by media (Baba and Hincal, 2018; Collinson and Heffernan, 2014; Collinson et al., 2015; Kumar et al., 2017; Misra et al., 2011; Samanta and Chattopadhyay, 2014; Samanta et al., 2013; Tchuenche and Bauch, 2012; Tchuenche et al., 2011; Wang et al., 2015). In recent years, a wide range of seminal papers had highlighted that information coverage may generate two different types of response (or feedback (FB)) with the outbreak of an infectious diseases. A first type of FB is the one given by the influence of the information on the behavior of health subjects (Capasso and Serio, 1978; D'Onofrio and Manfredi, 2009; Misra et al., 2015; Vargas-De-Leon and D'Onofrio, 2017); A second type of FB is the pseudo-rational exemption which is defined as the family's decision to not vaccinate children because of a pseudo-ration comparison between the perceived risk of infection and the perceived risk side effects caused by the vaccine (Buonomo et al., 2008; D'Onofrio et al., 2011). Therefore, it can help guide more effective reporting of disease cases and strategies on containing infectious prevalence to have a good knowledge of the effects of behavior changes with information influenced.

From the 1970s, the first epidemiological model dealing with behavioral changes was proposed by Capasso and Serio (1978). Capasso and Serio transformed the contact rate to be a decreasing function of the disease prevalence (i.e. the infection fraction in the total population), which implies that the size of infectious individuals would be regarded as an information. This would trigger control measures to reduce the rate of infection. Subsequently, several other works have been devoted to epidemic models with information-related contact pattern (Achieng et al., 2016; Mitchell and Ross, 2016; Xiao et al., 2015). There are two main approaches to represent contact rate in the form of a saturated function (Berrhazi et al., 2017; Kassa and Ouhinou, 2011) or a gamma distributed delay summarizing the current and the past history of the disease prevalence (Buonomo et al., 2008; D'Onofrio

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and Manfredi, 2009). Another option is to explicitly subdivide susceptible compartment into aware and unaware part, which provides insights into how does the transmission of awareness affect the disease spreading in the population (Agaba et al., 2017a; 2017c).

On the other hand, few works have been carried out the impact of behavioral changes on the age-structured dynamical systems. There is a recurrent phenomenon that the latent or incomplete treatment may revert back to the infectious class, also called reactivation, such as herpes (van den Driessche and Zou, 2007), TB (Castillo-Chavez and Feng, 1998; Li et al., 2017; Liu et al., 2015). Naturally, the latent phase and relapse phase ought to be considered into the model of infectious disease. Recent studies have established age-dependent latent and relapse models (Cao et al., 2017; Duan et al., 2017; Kuniya, 2011; Li et al., 2017; Wang et al., 2016) and studied the influence of latent and relapse age on stability of equilibria. Therefore, it is more practical to take into consideration the factor of age-dependent latency and relapse to describe certain disease spreading.

We concern with the first type of feedback in an age-dependent latent and relapse model. We formulate an SEIR-type epidemic model with behavioral change in contact pattern. The contact pattern is a function of some information index $M(t)$ due to the past and the present of disease states. $M(t)$ is provided through a suitable function ϱ_α^n as follows (Vargas-De-Leon and D'Onofrio, 2017):

$$M(t) = \int_{-\infty}^{t} g(I(\tau))\varrho_\alpha^n(t-\tau)d\tau. \tag{1}$$

As for g , a linear form $g(I) = \omega I$ was proposed in D'Onofrio and Manfredi (2009), Vargas-De-Leon and D'Onofrio (2017), where ω depends on pathogenicity. It can also be saturated, such as $g(I) = \omega I/(1 + qI)$, where ω and q are positive constants. Here we define $g(I)$ as a general function. The weight of past prevalence is represented by a delayed kernel ϱ_α^n. Generally, ϱ_α^n is the density function for a gamma distribution:

$$\varrho_\alpha^n(u) = \frac{\alpha^{n+1}u^n}{n!}e^{-\alpha u},$$

with a constant $\alpha > 0$ and an integer $n \geq 0$. α is assumed to be the average length of the historical memory concerning the disease in study, and the biological meaning of inverse the average delay of the collected information on the disease. The average delay is defined by $\tau = (n + 1)/\alpha$, and n is called the order of the delay kernel.

This paper is organized as follows. In Section 2, we formulate an *SEIR* information-related and age-dependent epidemic model, and define several useful notations. In Section 3, some preliminary properties of the *SEIR* model are presented. In Section 4, we commit to asymptotic smoothness of the semi-flow generated by the system. In Section 5, we investigate the existence of equilibria and propose the basic reproduction number $\Re_0$. The core of this paper is given in Section 6 which shows the uniform persistence, local stability and global stability of equilibria. Finally, some numerical simulations and conclusions are provided in Section 7 and Section 8, respectively.

2. Model formulation

Liu et al. (2015) investigated an age-structured model with the following form

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - \mu S(t) - \beta S(t)I(t), \\ \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial a}\right)e(t, a) = -\sigma(a)e(t, a) - \mu e(t, a), \\ \dfrac{dI(t)}{dt} = \int_0^\infty \sigma(a)e(t, a)da - (\mu + k)I(t) + \int_0^\infty \gamma(b)r(t, b)db, \\ \left(\dfrac{\partial}{\partial t} + \dfrac{\partial}{\partial b}\right)r(t, b) = -\gamma(b)r(t, b) - \mu r(t, b), \end{cases} \tag{2}$$

with the boundary conditions

$$e(t, 0) = \beta S(t)I(t), \quad r(t, 0) = kI(t)$$

for $t \geq 0$ and the initial conditions

$$S(0) = S_0, \quad e(0, a) = e_0(a), \quad I(0) = I_0, \quad r(0, b) = r_0(b)$$

for $a, b \geq 0$. Here, $S(t)$ and $I(t)$ denote the densities of susceptible individuals and infectious individuals at time t, respectively. $e(t, a)$ and $r(t, b)$ denote the densities at the time t of latent individuals with latent age a and removed individuals with relapse age b, respectively. $\sigma(a)$ and $\gamma(b)$ denote the conversional rate from latent class and the relapse rate in removed class, which depend on the age a and b, respectively. It is assumed that β is the transmission rate of the disease between susceptible and infectious individuals, μ is the natural death or birth rate, and k is the recovery rate from the infectious class. All parameters are assumed to be positive. System (2) exhibits the threshold phenomenon determined by the basic reproduction number.

Vargas-De-Leon and D'Onofrio (2017) considered the information index into an SEIR epidemic model, in which the contact function hinges upon some information index $M(t)$ denoted as the cumulative density of message about the past and the present of disease prevalence. And $M(t)$ is provided through an appropriate function (1).

As the widespread impact of information on epidemic, it is practically significant and applicable to consider information into age-structured epidemic models. Then, we adopt the expression of information in accordance with Eq. (1) for first type of feedback. Therefore, we construct the following epidemic model for a nonfatal disease with information-related contact rate and age-dependent latency and relapse, as well as give the schematic flowchart in Fig. 1.

$$\begin{cases} \frac{dS(t)}{dt} = \mu(1 - S(t)) - \beta(M(t))S(t)I(t), \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right)e(t, a) = -\sigma(a)e(t, a) - \mu e(t, a), \\ \frac{dI}{dt} = \int_0^\infty \sigma(a)e(t, a)da - (\mu + k)I(t) + \int_0^\infty \gamma(b)r(t, b)db, \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial b}\right)r(t, b) = -\gamma(b)r(t, b) - \mu r(t, b), \\ \frac{dM}{dt} = \alpha g(I) - \alpha M, \end{cases} \tag{3}$$

with the boundary conditions

$$e(t, 0) = \beta(M(t))S(t)I(t), \quad r(t, 0) = kI(t), \quad \text{for} \quad t > 0. \tag{4}$$

The initial conditions are given as

$$S(0) = S_0, \quad e(0, a) = e_0(a), \quad I(0) = I_0, \quad r(0, b) = r_0(b), \quad M(0) = M_0, \tag{5}$$

for $a, b \geq 0$. Here $S_0, I_0, M_0 \in \mathbb{R}^+$ and the densities of latent individuals and removed individuals at the initial time $e_0(a), r_0(b) \in L^1_{(0,+\infty)}$, which is the space of nonnegative and Lebesgue integrable functions on $[0, +\infty)$. Contact rate $\beta(M)$ describes the negative feedback of susceptible individuals about infectious spread, as a decreasing function with respect to $M(t)$. Throughout this paper, we use the kernel with $n = 0$ for information index $M(t)$, as a case of weak exponentially fading memory. Thus, $M(t)$ satisfies,

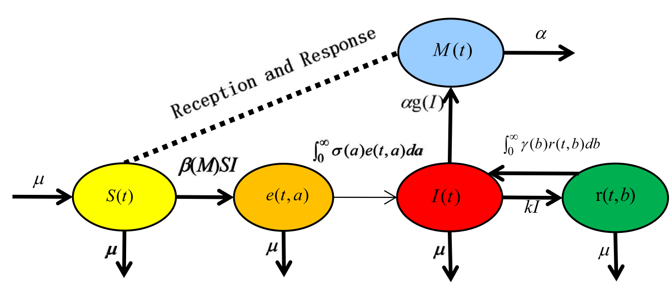

Fig. 1. Flowchart for system (3) with the impact of information.