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Decreasing fractal dimensions as a strategy for oceanic wildlife conservation: Application to species with large migration patterns

J.C. Flores

Instituto de Alta Investigación IAI, Universidad de Tarapacá, Casilla 7-D, Arica, Chile

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ABSTRACT

Wildlife dispersion patterns are responses of populations confronting variable environmental conditions. Additionally, industry depredation in oceans develops spatial patterns to optimize wildlife capture. Soft scaling conditions between protected and exploited marine zones define operative fractional dimensions for industry and wildlife. When reduction of the fractional dimension of industry ship trajectories is suitably established, the chances for wildlife to survive are increased. Accordingly, a protection strategy is proposed focusing on trajectory patterns rather than uniform areas. As a specific case, narwhals (*Monodon monoceros*) in the Arctic are considered. This approach best suits species with large-scale migratory patterns. Parameters are evaluated using current oceanic data.

1. Introduction

Complex spatial patterns can be partially mimicked through nonlinear models of differential equations, without spatial variables, which display scaling laws (Barraquand and Murrell, 2013; Hassell and May, 1974; Morozov and Poggiale, 2012; Pascual et al., 2011). This operational strategy is called mean-field dynamics. Nevertheless, as occurs in critical phenomena (Le Bellac, 1992), this approach does not allow capture of fine detail in ecology, such as large-scale spatial correlations and fluctuations (Baurmann et al., 2007; Bergstrom et al., 2006; Wang et al., 2007).

Moreover, scaling mathematical tools are widely used in ecology, for instance, helping to better understand the interaction between phytoplankton and zooplankton grazers in water columns (Cordoleani et al., 2013). These tools have also been used to study species migration in the Holocene (Lischke, 2005) and temperature mass-scaling for marine invertebrates (Barneche et al., 2016). Ferguson et al. (1998) dealt with fractal dimension evaluations for polar bear mobility. Mouillot and Viale (2001) estimated a low fractal dimension for fin whales (see also Cotté et al., 2009) and, additionally, discussed the role of Coriolis force on the trajectory (α -trajectories). Turchin (1996); Russell et al. (1992); Doerr and Doerr (2004), and Halley et al. (2004) considered fractal dimensions in ecology. Specific values for marine mammals were provided in the review by Seuront and Cribb (2017) and by Fleming (2016) for telemetry path reconstructions. Weinberger et al. (2017) develop mathematical tools concerning innovation, ecological stability and growth of human population and, Frank et al., 2018 develop an Anthropocene view for depredation and, eventual, extinction.

Regarding diversity of species and habitats, the effectiveness of uniform area protection has been questioned (Claudet et al., 2008; Mumby et al., 2011; Rodrigues et al., 2004). In fact, large specimens like whales, tunas, and dolphins exhibit large-scale global migrations or, at least, live in large regions, as do, for example, bottlenose dolphins (*Tursiops truncatus*) in north-central Chile. In this work, a strategy based on traveling patterns rather than uniform areas for wildlife protection is proposed.

Let P and U be the effective zone fractions of protected and unprotected oceans, respectively. Define R as the fraction of the depredated ocean zone (removed biota) resulting from generic industry exploitation, or hunting processes. Moreover, adopt unprotected zone disappearance $U = 0$ as defining the attractor (fixed point) of the ecosystem. As oceans are finite, for any time t, there is the constraint

$$P + U + R = 1 \tag{1}$$

Additionally, assume the dynamical scaling law for relative rates:

$$\frac{1}{rP}\frac{dP}{dt} = \frac{1}{\alpha R}\frac{dR}{dt} \tag{2}$$

The rate parameter r becomes related to generic social accords against industry overexploitation; it is a rate measuring the physical advance of protected zones. In contrast, the rate α refers to industry annexation of zones for over-exploitation. Concerning Eq. (2), there is a public attitude related to protection and, on the other hand, human requirements associated with depredation. Namely, there are social pressures on both types of organizations connected to P and R. In this sense, dynamical scaling law Eq. (2) assumes that these pressures are, in

E-mail address: cflores.iai@gmail.com.

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absolute terms, equivalents.

As an example of such structures, consider the coupled system of differential equations:

$$\frac{dP}{dt} = rPU \times G(P, R)$$

$$\frac{dU}{dt} = -(rP + \alpha R) \times U \times G(P, R)$$

$$\frac{dR}{dt} = \alpha RU \times G(P, R),$$

where *G(P,R)* is a regular generic function. The above set satisfies requirements (1) and (2). Finally, the technical analysis carried out in this paper also applies when equality in Eq. (2) is relaxed to be an approximation (soft scaling).

Finally, other fixed points ($U \neq 0$) could be considered as attractors by relaxing Eq. (1) as $P + U + R < 1$. Or by putting a different initial condition formally on *P,* containing the new value of *U*. In any case, only by simplicity in this article, the asymptotic extreme case $U = 0$ is considered.

2. Protected and removed fractional dimensions: x, y

Convoluted and complex paths can be related to fractional dimensions and, in ecology, wildlife trajectory patterns are interconnected with the structural complexity of habitats (Murray, 2002; Turchin, 1996). Consider a habitat with spatial integer (co-) dimension d, usually $d = 2$ for ocean surfaces. Let N^d, with N as the integer, be the number of spatial patches covering the habitat. Furthermore, consider $P_o = 1/N^d$ and $R_o = 1/N^d$ as the initial conditions for ecological systems described by Eqs. (1) and (2).

Define the fractional dimension related to the protected and removed zones, x and y, respectively, through

$$P = \frac{N^x}{N^d} \text{ and } R = \frac{N^y}{N^d} \tag{3}$$

Note that re-definitions $M = N^d$ and $P = (N/a)^x$ allow the usual meaning of fractal dimension $x = \ln(M(a))/\ln(a)$. Similarly for dimension y.

In the point of equilibrium (attractor), Eqs. (1) and (2) give the relationships

$$\frac{N^x}{N^d} + \frac{N^y}{N^d} = 1 \text{ and } \frac{x}{r} = \frac{y}{\alpha} \tag{4}$$

which are regarded as the underlying equations in this work (generalization provided in Section 4, Eq. (8)). They define complex spatial patterns through the fractional dimensions x and y. Fractional dimensions are assumed to be almost invariant in the appropriate oceanic ranges: for a practical example see Mouillot and Viale (2001). In this work, we consider explicitly the oceanic surface case where dimensions x and y are not larger than two.

3. Oceanic parameter estimations and narwhal patterns

Separating Eq. (4) for dimensions x and y yields

$$\frac{\alpha}{r} = \frac{1}{x\ln(N)}\ln(N^d - N^x) \tag{5}$$

$$\frac{\alpha}{r} = y\ln(N)\frac{1}{\ln(N^d - N^y)} \tag{6}$$

Fig. 1 shows the ratio α/r as a function of the protected fractional dimension x (green curve) and the removed biota y (red curve) with the number of patches set to 10^2. The superior inset graphic represents the limit $N \to \infty$, showing that the conclusions are not too sensitive to the number of spots provided that $N \gg 1$. On the graph, note that for x and y the maximal dimension value is $d = 2$. It is a theoretical value since no vessel or animal reach these extreme patterns. In fact, Fig. 1 describes the worse scenario for wildlife.

The red hatched region between the curves represents the significant risk for wildlife, where the fractional dimension of the depredation zone is larger than the protected one ($x < y$, or $P < R$), corresponding to the well-known tragedy of the commons (De Young and Kaplan, 1988; Edney and Harper, 1978; Hardin, 1968). The green hatched region is the most appropriate for safety ($y < x$, or $R < P$).

Note that formally the third dimension (ocean depth) is included in Fig. 1, assuming by the simplicity that it remains unaffected by industry depredation and prey do not hide there.

Globally, from 1990 to 2014, marine areas within national jurisdiction grew from 1% to 8.4% of the global ocean area (UNEP-WCMC, at 2014). As an example, more than 463,000 km^2 of marine zones are today protected in Chile (Petit et al., 2018). Thus, a global value $r \approx (8.4\text{–}1)/(8.4 \times 24) \approx 0.036$ (1/yr.) seems acceptable. In contrast, in the last 50 years, the big-animal populations of the world's oceans have been reduced by around 90% (Jackson, 2008; Myers and Worm, 2003, 2005). Assuming that the removed zone grows exponentially, the corresponding estimate of the industry annexation rate is thus $\alpha_1 \approx 0.046$ (1/yr.). Darimont et al. (2015) gave a value for recent years of $\alpha_2 \approx 0.17$ (1/yr); consequently, an intermediate estimate of $\alpha \approx 0.1$ seems acceptable. Accordingly, the annexations ratio can be obtained:

$$\frac{\alpha}{r} \approx 2.7, \tag{7}$$

corresponding to the horizontal dashed blue line in Fig. 1. This sensible value can change because of other estimations for the annexation parameters or fluctuations; nevertheless, the conclusions do not differ significantly providing $\alpha > r$, in accord with the current decimation context (Darimont et al., 2015; Isbell et al., 2015).

Consider the case of narwhals (*Monodon monoceros*) in West Greenland and the eastern Canadian High Arctic. Kristin et al. (2004) calculated the fractal dimensions for groups of narwhals, using satellite tracking. They found that fractal dimension is seasonal, with mean average values of 1.61 in summer, 1.69 in winter, and 1.34 when narwhals are migrating (black dots in Fig. 1). This result corresponds to a two-phase scheme called exploitation/relocation (Bénichou et al., 2011; Flores, 2013) assigning, additionally in the case of Kristin et al. (2004), fractal dimension estimations. The black dashed rectangle in Fig. 1 defines the narwhals' region for different values of the parameter α/r and fractal dimension in this worse scenario. Importantly, the value for $\alpha/r = 2.7$ (blue line) and the range of narwhal fractal dimension intersect clearly in the overexploitation zone (red hatching), indicating risk of narwhal decimation (Jefferson et al., 2012).

4. Decreasing industry fractional dimension (hunting patterns) as a strategy for preservation

Fig. 1 shows the situation for generic ecosystems described by Eqs. (1) and (2) or, equivalently, Eq. (4). Facing depredation machinery, wildlife has more options when the entire overexploitation zone (red) is reduced or correctly shifted. To this end, two approaches exist: strategy (a) is the conventional one; strategy (b) is innovative.

(a) To diminish the ratio α/r (horizontal blue line in Fig. 1) requires enhancing the protection zones, currently 8.4% for marine areas, by sustained growth of the rate r. Alternatively, reduction of the depredation rate α requires substantial international agreements (Donohue et al., 2016).

(b) A novel strategy is to reduce the fractional dimension associated with industry exploitation patterns (hunter trajectories). In fact, for the industry, the maximal fractional dimension allowed in the solution of Eqs. (1) and (2) is $y = d$ assuming $d = 2$ for ocean surfaces in Fig. 1. Consider that industry depredation is forced to a maximal fractional dimension d' smaller than d. In this case, the first