Ecological Modelling 381 (2018) 36–45

Contents lists available at ScienceDirect

Ecological Modelling

journal homepage: www.elsevier.com/locate/ecolmodel

Management of invasive insect species using optimal control theory

Christina J. Edholm[a,b,*], Brigitte Tenhumberg[b,c], Chris Guiver[d], Yu Jin[b], Stuart Townley[e], Richard Rebarber[b]

[a] Department of Mathematics, University of Tennessee, Knoxville, Knoxville, TN 37916, United States
[b] Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588, United States
[c] School of Biological Sciences, University of Nebraska-Lincoln, Lincoln, NE 68588, United States
[d] Department of Mathematical Sciences, University of Bath, Bath, UK
[e] Environment and Sustainability Institute, University of Exeter, Penryn, Cornwall, UK

ARTICLE INFO

Keywords:
Population management
Optimal control
Invasive species
Discrete-time system
Diaprepes abbreviatus

ABSTRACT

We discuss the use of optimal control theory to determine the most cost-effective management strategies for insect pests. We use a stage-structured linear population projection model where the modeled control action increases the mortality in one of the stage-classes. We illustrate the method by using a published model for the root weevil *Diaprepes abbreviatus*, an invasive insect species having a substantial negative impact on citrus trees in regions such as Florida and California in the United States. Here control corresponds to the application of inundative biological control agents (entomopathogenic nematodes as biopesticides) which increases the mortality of the larval stage. Our approach determines levels and timing of control to minimize the economic loss caused by *D. abbreviatus*. We use two numerical methods to approximate the optimal control, and compare their effectiveness.

1. Introduction

Insect pests cause considerable economic loss to agriculture worldwide. This economic loss consists of reduced harvest and the cost of applying pesticide or biological control. The costs of pest control has been steadily increasing. For instance, from 2008–2012 the insecticide expenditure for producers increased worldwide from 12.5 to 16 billion US Dollars, and in the USA from 22 to 25 million US Dollars (Atwood and Paisley-Jones, 2017). Further, pesticide usage incurs indirect environmental and economic costs associated with the recommended application of pesticides. These indirect costs include pesticide poisonings and illnesses of humans, domestic animals, and negative impact on beneficial animals (Pimentel and Burgess, 2014). In most cases the extensive use of pesticides also leads to the evolution of pesticide resistance which often necessitates an increase in insecticide application. As a response to these challenges some producers turn to using biological control methods including the release of natural enemies of the pest insects such as predators, parasitoids or pathogens (Ehler, 1998; Pell et al., 2010; Seastedt, 2014). Establishing a sufficiently high population of natural enemies can be challenging because of negative effects of disturbances caused by agronomic interventions such as tilling and the general low quality and diversity of agricultural landscapes. However, natural enemies can still be effective if they temporarily reach high numbers via augmentations. In this case, they function more like a pesticide that persists in the environment for a limited amount of time without any of the negative effects of chemical pesticides.

In this paper we present a mathematical framework for designing a management strategy that maximizes the profit for producers by using pest control strategies most effectively. We apply our framework to the management of the economically important citrus fruit insect pest, *Diaprepes abbreviates*, commonly referred to as Diaprepes root weevils, DRW (Grafton-Cardwell, 2004). DRW is a long-lived invasive insect species, which is native to the Caribbean. It has been established in Florida and California (Grafton-Cardwell, 2004; Jetter and Godfrey, 2009 Jetter and Godfrey, 2009). This insect species has four distinct life history stages (eggs, larva, pupae, and adults) of which the larvae are the by far most damaging stage. One promising strategy for targeting the larva stage is the application of commercially available entomopathogenic nematodes for inundative biological control (i.e., as biopesticides) (Shapiro-Ilan et al., 2002; Bullock et al., 1999). Entomopathogenic nematodes are parasites of insects that kill the infected host usually within 24–48 h. They can be applied with most horticultural equipment including pressurized sprayers, mist blowers, and electrostatic sprayers, and are used as biopesticide for a range of different pest insects (Georgis et al., 2006). While there has been research

* Corresponding author at: Department of Mathematics, University of Tennessee, Knoxville, Knoxville, TN 37916, United States.
E-mail address: cedholm@utk.edu (C.J. Edholm).

https://doi.org/10.1016/j.ecolmodel.2018.04.011
Received 22 October 2017; Received in revised form 18 April 2018; Accepted 19 April 2018
0304-3800/

on DRW (Grafton-Cardwell, 2004; Jetter and Godfrey, 2009 Jetter and Godfrey, 2009), to the authors' knowledge this is the first use of optimal control theory to consider management options for DRW.

Specifically, we explore the use of optimal control theory for pest management, continuing a line of inquiry into optimal control in biological applications. Optimal control theory broadly refers to the area of mathematics and engineering where a control action is determined to achieve some desired dynamic behavior *and* minimize a prescribed cost functional. In the context of pest management, the desired dynamic behavior is a reduction in pest abundance whilst the cost functional models the combined cost of the loss of crop to pest and application of control. The reader is referred to Macki and Strauss (2012) and Filippov (1962) for more background on optimal control theory, and to Lenhart and Workman (2007) for optimal control and its biological applications. The papers Hastings et al. (2006); Blackwood et al. (2010) and Lampert et al. (2014) outline general models for invasive species, specifically considering the invasive grass species, *Spartina alterniflora*. The paper Hastings et al. (2006) utilizes linear programming, while Blackwood et al. (2010) implements linear-quadratic control with a spatially-explicit model using dynamic programming. The paper Lampert et al. (2014) incorporates endangered species into the model along with the invasive species. The papers Dabbs (2010) and Whittle et al. (2008, 2007) each model invasive species and use discrete optimal control theory to analyze the systems. In Dabbs (2010), part of the population has a nonlinear growth function, and they compare the impact of different choices of growth functions on the optimal control. The paper Whittle et al. (2008) formulates a discrete time optimal control problem for Gypsy Moths adding a control linearly into the system, where the control they are adding is also already naturally in the environment. Lastly, Whittle et al. (2007) considers discrete optimal control problems for invasive plant species, including budgetary constraints.

We propose a linear, stage-structured population model in discrete time representing a generalized life cycle of an insect species (Caswell, 2001) and consider a control action that reduces survival of a single stage-class. For our model, the control is incorporated into the state equations through a nonlinear function. This type of control could apply to biopesticides like entomopathogenic nematodes or chemical insecticide applications using products affecting a single stage-class. For instance, for species where the larval stage lives in the soil, toxins may be sprayed on the surface and kill newly hatched larvae while burying into the soil (Shapiro-Ilan et al., 2002). We consider two different types of control. First, we assume that each control action affects the insects for a single time-step only. Second, we consider control which persists, but decays exponentially over time post application. We define a cost functional to describe loss in harvest income from the pest plus the cost of applying the control. We say that a control strategy is optimal if it minimizes the cost functional subject to the dynamics of the controlled pest population. The optimal amount and timing of control application should balance the cost of the control with the cost of allowing the pest to reduce the harvest. The resulting optimal control is likely to be time dependent. For instance, it might be best to start applying control when pest density is low and keep applying control over the entire season (frequent control at low intensity). Alternatively, it might be better to carry out a small number of control actions with high intensity and allow pest densities to build up between control actions. Optimal control techniques can help decide between competing control schemes, even if the optimal control is difficult to apply.

We compare two different numerical search algorithms using MATLAB to solve the optimal control problem, since finding the optimal controls analytically is typically intractable. The Forward-Backward Sweep is a very efficient method for approximating optimal controls and relies on the Pontryagin Maximum Principle (Pontryagin et al., 1962), via a dynamical system called the adjoint system (Lenhart and Workman, 2007, Chp. 4, 23). However, we will see that this method breaks down if the control decays slowly over time. In our second method we implement a standard MATLAB function called MultiStart, which does not rely on the adjoint system, but is much more computationally intensive. With enough computational runs MultiStart produces a reasonable solution, but is less accurate than Forward-Backward Sweep.

This paper is organized as follows. In Section 2, we formulate the optimal control problem for a model for a general insect species with control, specify a cost function, and establish the existence of solutions to the optimal control problems. We illustrate our method with a case study on *Diaprepes abbreviatus*, DRW (Grafton-Cardwell, 2004). In Section 3, we give numerical simulations of our DRW case study. We discuss our results and their significance in Section 4.

2. Methods

2.1. Model formulation

We start this section with a matrix model for the dynamics of an invasive insect pest. We model the pest in four distinct stage-classes, denoted by $P_1(t)$, $P_2(t)$, $P_3(t)$ and $P_4(t)$, at time-step t. Here the time variable $t \in \mathbb{Z}_+ = \{0, 1, 2, \ldots\}$ denotes how many time-steps have passed. We denote the four-dimensional vector with these stages by

$$\mathbf{P}(t) := [P_1(t) \quad P_2(t) \quad P_3(t) \quad P_4(t)]^\top, \quad t \in \mathbb{Z}_+,$$

where the superscript T denotes vector transposition. All of the models we consider are local in the sense that there is no explicit spatial dependence or variation, only temporal. For the applications we have in mind, the stage-classes shall denote the abundance of the four distinct developmental stages: eggs, larvae, pupae and adults. The associated population projection matrix is the 4×4 matrix:

$$A = \begin{bmatrix} \gamma_1 & 0 & 0 & \theta_1 \\ \gamma_2 & \zeta_1 & 0 & 0 \\ 0 & \zeta_2 & \nu_1 & 0 \\ 0 & 0 & \nu_2 & \theta_2 \end{bmatrix},$$

where θ_j, γ_j, ν_j, and ζ_j are nonnegative parameters, for $j \in \{1, 2\}$. The descriptions for the matrix A parameters are in Table (2.1). We assume that a scalar control, denoted by $N(t)$, is applied at each time-step. We consider the situation where the control is applied to only one stage-class; here we choose the second stage-class P_2, but it can be applied to any stage. To capture saturation effects reflecting diminishing returns for large control efforts, we assume that the control efficacy is given by $f(N(t))$, for a given function $f: \mathbb{R}_+ \to \mathbb{R}_+$. We typically assume that f is nonincreasing. The dynamics of the pest with control are described by:

$$\mathbf{P}(t+1) = \mathscr{A}(N(t))\mathbf{P}(t) \quad \mathbf{P}(0) = \mathbf{P}^0 \quad t \in \mathbb{Z}_+, \tag{2.1}$$

where

$$\mathscr{A}(N(t)) = \begin{bmatrix} \gamma_1 & 0\cdot f(N(t)) & 0 & \theta_1 \\ \gamma_2 & \zeta_1\cdot f(N(t)) & 0 & 0 \\ 0 & \zeta_2\cdot f(N(t)) & \nu_1 & 0 \\ 0 & 0\cdot f(N(t)) & \nu_2 & \theta_2 \end{bmatrix} \quad \text{and} \quad \mathbf{P}^0 = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix}$$

denote the nonlinear projection matrix and initial population, respectively. Note that the model counts the transitions from one population census to the next, where we count immediately after birth. Hence, the model assumes that the control does not impact the transition from P_1 to P_2. Fig. 2.1 depicts the model dynamics in Eq. (2.1).

We comment that the model (2.1), and the approach we take, generalizes to higher (but finite) numbers of stage-classes. Specifically, for $n \in \mathbb{Z}_+$, $\mathbf{P}$ can be replaced by

$$[P_1(t) \quad P_2(t) \quad \ldots \quad P_n(t)]^\top, \quad t \in \mathbb{Z}_+,$$

and A can be replaced by the $n \times n$ matrix