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Participation games: Market entry, coordination, and the beautiful blonde☆

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Abstract

We find the Nash equilibria for monotone n-player symmetric games where each player chooses whether to participate. Examples include market entry games, coordination games, and the bar-room game depicted in the movie "A Beautiful Mind". The symmetric Nash equilibrium involves excessive participation (a common property resource problem) if participants' payoffs are decreasing (in the number of participants), and insufficient participation if payoffs are increasing. With decreasing payoffs there can be many equilibria, but with increasing payoffs there are only 3. Some comparative static properties of changing one player's participation payoffs are counterintuitive, especially with more than two players.

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1. Introduction

Economists have studied several types of "participation game" in which each player chooses whether to participate in an activity and payoffs depend on the number of players who do so. Typically, participation games have a monotonicity property: payoffs either always decrease with the number of participants or always increase. A major class of examples in the field of industrial organization describes the decisions of firms on whether to enter a market. When firms compete to sell substitute products, payoffs typically decrease with the number of entrants. Less seriously,

☆ This is a revised version of "A beautiful blonde: a Nash coordination game."

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a scene from the recent movie about John Nash, “A Beautiful Mind,” describes the decision of a group of men whether to pursue a particular woman. This game has exactly the same structure as an entry game where post-entry market interaction is Bertrand competition with homogenous products. Important examples where participation payoffs increase with the number of participants include the adoption of innovations with positive network externalities (such as telephones or fax machines) and variations of the classic stag-hunt game originally due to the 18th-century French philosopher Jean-Jacques Rousseau.

Participation games typically exhibit pure strategy as well as mixed strategy equilibria. With a symmetric payoff function, there exists a symmetric mixed strategy equilibrium. When a player’s participation decreases the participation payoff of the others, the equilibrium involves excessive participation (from a social perspective) since each player is indifferent at such a mixed strategy between participating and not. This we view as a kind of common property resource problem associated with the mixed strategy equilibrium. We also find that there are typically semi-mixed equilibria at which some players play pure strategies and others randomize. By the reasoning above, they also involve excessive participation.

These mixed and semi-mixed strategy equilibria have some intriguing comparative static properties. For example, if the payoff to all players from participating rises, then (as one might expect) the new symmetric mixed strategy equilibrium involves all participating with higher probability.[1] This intuitive result masks a counter-intuitive property. Suppose we were to raise participation payoffs one at a time for players. Then, making the first player value participation more causes the other players to increase their participation probabilities to keep the first player indifferent. If there is just one other player, the player whose payoff rises has no change in participation probability, but the other player’s participation probability must rise.

With more than two players, the outcome is even stranger. The fact that the unaffected players’ probabilities must rise means that they are worse off participating than before the change, so to keep them indifferent (and so mixing), the affected player’s participation probability must actually fall! This means that the player who values participation higher must actually participate with *lower* probability. The others participate with higher probability in the new mixed strategy equilibrium. However, when all players’ benefit from participation rises, they all end up participating more frequently. This result seems “right for the wrong reason” in the sense that the result arises because each is participating more to keep the others indifferent.

Similar results apply to participation games for which payoffs instead rise with the number of participants. However, there are some essential differences between participation games with congestion (such as the market entry game) games with positive synergies (such as the stag-hunt). While these games exhibit a superficial similarity when there are two players, with more players the congestion games have more equilibria. Games with positive synergies either involve all players playing the same pure strategy or else all of them randomizing, so that these games have only three equilibria. Congestion games allow additional equilibria in which some, but not all, of the players randomize.

Before giving the general analysis, we first describe the main results in the simple and specific context of the game from the Nash movie. This enables us to provide the underlying intuition and setting, which we cover in Section 2. In Section 3, we provide our main results (in the context of the participation game with rivalries amongst entrants) on the set of equilibria, on the common

[1] A similar result holds for the equilibrium probabilities of each of the individuals who randomize at each of the semi-mixed equilibria.