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Modeling a large population of traders: Mimesis and stability[☆]

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Abstract

We introduce a method of accurately and efficiently modeling a large population of participants in a financial market. Each participant is modeled as having an internal preference state affected by the continual arrival of exogenous information and by the behavior of others. In order to describe a community of traders, we introduce a population equation that is derived rigorously from the underlying single-agent model. The population equation is used to investigate collective behavior with mimetic interactions. We observe and study the sharp transitions in parameter space from a stable time-independent regime to instability where the demand and supply diverge sharply.

JEL classification: C63; D79; G10

Keywords: Herd behavior; Mimesis; Collective decision making; Financial markets; Stability

1. Introduction

Behavioral traits of market participants and their effects on the properties of markets have been under growing investigation in recent years. Much of this work (e.g., Arthur et al., 1997; Challet and Zhang, 1997; Chan et al., 1998; Lux and Marchesi, 1999; Shiller, 1999; Cont and Bouchaud, 2000; Farmer, 2002; Iori, 2002), which emphasizes deviations from efficient markets theory, tries to elicit an internal market dynamics via computational approaches and is loosely

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linked to the subject known as behavioral finance. These interdisciplinary studies try to explain market phenomena by formalizing a diverse array of behavioral and psychological considerations.

Despite the emphasis on the characteristics of real market participants, models of individual behavior in agent-based models have remained rather simple. An agent's psychology as distinct from manifest behavior is typically not separately represented in an agent-based model. Further, an agent is often modeled as a unit capable of being in one of a small number of states. It is generally believed that agent-based modeling is in its early stages (Farmer, 2001), but the simplicity of most existing agent models may also derive from a number of other (valid) reasons: (i) attempts to extract minimal conditions responsible for the "stylized facts" of price statistics (Cont, 2001); agent-based modeling has been very successful in this regard; (ii) computational costs and the difficulty of drawing insights from simulating a very large number of realistic agents and their interactions; these have become considerable especially with multiple agent types and interactions, so that methods beyond direct simulation need to be specified; and (iii) lack of well-accepted quantitative models of deliberation and the behaviors associated with it. In this study we try to address the last two items. In particular we develop a framework for analyzing and efficiently simulating large populations of participants in a financial market whose psychology and behavior can be specified with greater precision than so far allowed.

We will first introduce a single-agent model capable of describing the process of deliberation and how it leads to action. In our model, each agent is described by an internal preference state with an intrinsic dynamics and influenced by external events within or outside the population. Actions result when the agent's state crosses a predetermined threshold in state space. Specifically, we consider a population consisting of traders repeatedly deciding whether to buy or sell an asset. We next formulate a dynamical equation describing the entire population of agents. Analogues of this equation are referred to as kinetic equations in statistical mechanics. We derive the kinetic (or population) equation rigorously from the single-agent model with the addition of a reasonable statistical assumption and with *no free parameters*. We investigate the implications of the kinetic equation for the rates of buy and sell orders that result from a steady input of information.

In addition to information that arrives from the outside and generates exogenous shocks, a population is also affected by the endogenous exchange of information. In fact, direct interaction and personal communication can exert a powerful influence on the individuals in humans groups in general. The ability of groups to act cohesively and respond collectively to information may have had a high adaptive value in evolutionary history. Cohesive human behavior resulting from mimesis or herding has been studied extensively (see Shiller, 2000, chapter 8, for a review and references). While possibly associated with emotional or "irrational" drives, mimesis can also be interpreted as a rational strategy under conditions of imperfect information. Recent economic literature continues to emphasize complex informational influences on behavior and their potential to bring about unexpected patterns (Bannerjee, 1992; Topol, 1991; Bikhchandani et al., 1992; Kirman, 1993; Orléan, 1995). In the relatively simplified context of agent-based financial market models, mimesis has been proposed as a major contributor to the strongly non-Gaussian nature of short to middle term price changes and to excess volatility (Lux and Marchesi, 1999; Cont and Bouchaud, 2000; Iori, 2002).

As an application of our method we will use the kinetic equation to investigate the effects of mimetic behavior. For mimetic agents, the kinetic equation takes the form of a nonlinear partial differential equation having a kinship to the Boltzmann equation of kinetic theory (Cercignani, 1988). This formulation allows us to analyze stability boundaries in parameter space theoretically.