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A phenomenological relationship between vertical air motion and disdrometer derived *A-b* coefficients

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ABSTRACT

Using the well-known *Z-R* power law, $Z = AR^b$, *A-b* parameters derived from a single disdrometer are readily found and can provide useful information to study rainfall drop size distributions (DSDs). However, large variations in values are often seen when comparing *A-b* sets from various researchers. Values of b typically range from 1.25 to 1.55 for both stratiform and convective events. The values of A approximately fall into three groups: 150 to 200 for convective, 200 to 400 for stratiform, and 400 to 500 for convective. Computing the *A-b* parameters using the gamma DSD, coupled with a modified drop terminal velocity model, $v_D(D) = v_T(D) - w$, where D is drop diameter, $v_T(D)$ is still air drop terminal velocity, and w is an estimate of vertical velocity of the air well above the disdrometer, shows an interesting result. This model predicts three regions of A, corresponding to $w < 0$, $w = 0$, and $w > 0$. Additional models that incorporate a constant vertical air velocity are also investigated. *A-b* sets derived from a Joss-Waldvogel (JW) disdrometer and DSD data acquired near Athalassa, Cyprus, using selected 24-hour data sets from 2011 to 2014, are compared to the above models. The data is separated into two main groups: stratiform events defined by rainfall rates that did not exceed 10 mm h^{-1} at any time during the 24-hour period, and convective events defined by rainfall rates not flagged as stratiform. The convective rainfall is further separated into two groups: *A-b* pairs that fall to the left of the stratiform pairs and pairs that fall to the right. This procedure is repeated with data from other researchers that corresponds to seasonal averages. In all cases, the three vertical groupings of the *A-b* parameter plot seem to correlate to DSD simulations where various values of positive and negative vertical velocities are used.

1. Introduction

Research on precipitation advances our ability to understand the components of the water cycle and their respective underlying mechanisms. In this respect, improving precipitation observing methods and systems at both the global and local scale, will ultimately affect the quality of precipitation-related products employed in applications across all scales, with consequential improvements in hydrologic forecasting and water resources management (see Michaelides et al., 2009). Precipitation is characterized by its drop size distribution. All or certainly most precipitation observing systems are based on measuring some aspect of the DSD. Two common classes of observing systems are those that measure the DSD flux, including rain gauges and disdrometers, and those that measure the aerial DSD. Flux measurements can be expressed as a fractional DSD moment, dependent on the choice of raindrop terminal velocity model. If the terminal velocity is proportional to D^γ, rainfall rate R [m s^{-1}] is then equal to the $(3 + \gamma)^{\text{th}}$ moment (see Caracciolo et al., 2006). Weather radar measures reflectivity Z [mm^6 m^{-3}], proportional to the 6th moment of the DSD, while the less common optical extinction σ [m^{-1}] measures the DSD's 2nd moment (Atlas, 1953; Shipley et al., 1974).

Fundamentally, the drop size distribution dictates the behavior of Z and R; Z and R are related through the well-known *Z-R* power law, $Z = A\ R^b$, that uses two parameters, A and b. It could be argued that characterizing the *A-b* parameter space is not a useful pursuit since it must be assumed that the *Z-R* relationship is based on a simple two parameter power law. For example, Chapon et al. (2008) studied the *Z-R* relation using a ground based disdrometer and radar data in southern France. The *Z-R* relationships derived from this DSD dataset were found to be very diverse. However, the *A-b* parameter model is an intrinsic part of the US National Weather Service (NWS) radar system and forecasting strategy (Wilson and Brandes, 1979; Choy et al., 1996). In

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other parts of the world, Ochou et al. (2007) collected Joss-Waldvogel (JW) disdrometer data at 4 sites in western Africa. Using a log-normal distribution model, they derived 4 sets of *A*-*b* parameters based on long term averages at each of the 4 data collection sites.

It is not just the DSD (usually represented by $N(D)$, where D refers to the drop size) that is important in rainfall studies; it is the DSD flux or drop flux distribution (DFD) on a surface that must also be known. In a sense, the *Z*-*R* relation could be considered a DSD-DFD relation. The connection between these two quantities is the drop velocity function $v_D(D)$. Note that caution must be exercised in using $v_D(D)$ since it is a vector quantity but is often treated as a scalar function. In Cartesian coordinates, the vector drop velocity $\mathbf{v}_D(D)$ can be decomposed into still air drop terminal velocity $v_T(D)$ in the direction of gravity, vertical component of air motion w, and horizontal components of air motion:

$$\mathbf{v}_D(D) = u_x\,\hat{\mathbf{i}} + u_y\,\hat{\mathbf{j}} + (w - v_T(D))\,\hat{\mathbf{k}}, \tag{1}$$

where u_x and u_y are the orthogonal components of the horizontal air velocity u. In general, u and w are functions of time and position. The DFD is then the quantity $N(D)v_D(D)$.

A problem with Eq. (1) is that the terminal velocity term $v_T(D)$ is only time independent as shown when u and w are both constant, and enough time has elapsed after the start of a drop trajectory that the sum of the drop's external forces are zero. For the most part, Z, R, $N(D)$, and $v_D(D)$ are Eulerian quantities since they are measured at fixed points in space and are based on a distribution of particles. The approach taken in this work, is to employ Lagrangian particle trajectory modeling to resolve this problem. This is especially important when modeling the drop velocity at the surface where a rain gauge or disdrometer would be located.

The drop size distribution $N(D)$ has traditionally been modeled as an exponential function or gamma function. The gamma drop size distribution is represented by three parameters, namely, μ, N_0, and Λ (Ulbrich, 1983) as follows:

$$N(D) = D^{\mu} N_0 e^{-\Lambda D}. \tag{2}$$

The gamma distribution reduces to the exponential distribution for $\mu = 0$. Numerous researchers have investigated correlations between the DSD shape and observable rainfall characteristics and processes (Rigby et al., 1954; Thurai et al., 2014). Some researchers have investigated relationships between the gamma distribution shape factor and physical processes such as coalescence (Hardy, 1962) and supersaturated updrafts (Igel and van den Heever, 2017).

Segregating rain types by means of disdrometer data has long been an active area of research. Using a normalized gamma distribution model, Marzano et al. (2010) investigated the latitude dependence of stratiform and convective rain types, as well as wet and dry periods using a JW disdrometer along with multi-frequency microwave radiometers and microwave polarimetric radar. Islam et al. (2012) using 7 years of JW disdrometer processed by a normalized gamma model, investigated warm, cold, wet, and dry rain types in the southern UK. More recently, Thurai et al. (2016) developed a robust stratiform-convective identification algorithm using two dimensions video disdrometer (2DVD) data, which was tested at sites in Ontario and Huntsville Alabama.

The main focus of this work is to incorporate w into a model of the DSD and DFD in order to observe the predicted effects on A and b and compare to the disdrometer derived A and b coefficients. Several models will be put forward, each with some advantages and disadvantages; these models are compared to the other. In the end, it will be demonstrated that there is a plausible and moderately predictable connection between the disdrometer derived *A*-*b* pair and the rainfall type, from which the sign and value of w can be estimated.

This paper is structured as follows: In Section 2, an overview of the traditional Marshall-Palmer DSD is given, in order to set the scene for the sections that follow. Section 3 presents a set of three DSD models, built sequentially, where the vertical component of air motion is incorporated into the raindrop terminal velocity and the drop size distribution. The results of the model outputs are compared to disdrometer derived *A*-*b* parameter pairs. Section 4 discusses the correlation between the simulation and disdrometer data and shows how the physics-based DSD models of Section 3 compare to the empirical gamma DSD model. Section 5 summarizes the overall results and discusses the successes and failures of each of the three models and associated data analysis; recommendations for future work are included in this section too.

2. The Marshall-Palmer DSD

The Marshall-Palmer (MP) DSD (Marshall and Palmer, 1948) is a special case of the *exponential* DSD. Because of the specific form of the MP DSD, consistency in computing rainfall rate from the DSD dictates a very specific form of the drop terminal velocity function which is a power-law described by two parameters, $v(D) = v_0 D^{\gamma}\,\mathrm{m\ s^{-1}}$ (with D in mm). Using this $v(D)$, and starting with $N(D)$ in Eq. (2) with $\mu = 0$, the reflectivity Z (for S-band and lower frequency bands where non-Rayleigh scattering effects are negligible) and rainfall rate R can be computed as follows:

$$\begin{aligned} Z &= \int_0^{\infty} D^6 N(D)\,dD \\ &= N_0\,\Gamma(7)\,\Lambda^{-7} \quad [\mathrm{m^{-3}mm^6}] \end{aligned}, \tag{3}$$

$$\begin{aligned} R &= a_R \int_0^{\infty} v(D) D^3 N(D)\,dD \\ &= v_0 N_0\,\Gamma(4+\gamma)\,\Lambda^{-4-\gamma} \quad [\mathrm{m\ s^{-1}}] \end{aligned}, \\ R = a_R v_0 N_0\,\Gamma(4+\gamma)\,\Lambda^{-4-\gamma} \quad [\mathrm{mm\ h^{-1}}] \tag{4}$$

where a_R is a units conversion constant such that R is represented in standard units of $\mathrm{mm\ h^{-1}}$: $a_R = 0.0036\,\pi/6$ (the $\pi/6$ is due to the drop volume geometry factor). Both Z and R are proportional to N_0 [$\mathrm{m^{-3}\,mm^{-1}}$], the *y*-intercept of the DSD model. In contrast, Z and R are inversely proportional to specific powers of the DSD slope parameter Λ [$\mathrm{mm^{-1}}$], where the exponent is equal to the DSD moment + 1.

By convention, the *Z*-*R* relation is also a power-law of the form $Z = AR^b$. A particular choice of b is independent of the DSD variables N_0 and Λ, which also leads to the solution of A:

$$b = \frac{7}{4+\gamma}, \tag{5}$$

$$A = N_0^{1-b}\,\Gamma(7)\,(a_R v_0\,\Gamma(4+\gamma))^{-b}. \tag{6}$$

Eqs. (3) through (6) are general results for the exponential DSD with a power-law terminal velocity function. Also, these results are dependent on the limits of integration, which assumes that raindrop diameter extends from 0 to ∞. A refined approach would incorporate a more realistic size range such as 0 to 6 mm. For an impact disdrometer, a size range would be determined by the limits of the instrument detection such as 0.3 mm to 5.5 mm in the case of the JW disdrometer.

The MP DSD defines the rate variable Λ as a function of R: $\Lambda(R) = \alpha R^{\beta}$, where $\alpha = 4.1$ and $\beta = -0.21$, so that Λ has units of $\mathrm{mm^{-1}}$ and R is expressed in $\mathrm{mm\ h^{-1}}$. Substituting this expression for Λ in Eq. (4) leads to the following:

$$\gamma = -\frac{1+4\beta}{\beta} = 0.762, \tag{7}$$

$$v_0 = \frac{\alpha^{-1/\beta}}{a_R N_0\,\Gamma(-1/\beta)} = 3.254. \tag{8}$$

Using these values, with the MP value of $N_0 = 8000\,\mathrm{m^{-3}\,mm^{-1}}$, Eqs. (5) and (6) evaluate to $b = 1.47$ and $A = 296$. Note that b is independent of Λ and N_0, while A is also independent of Λ but dependent on N_0.