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A unified solution for vibration analysis of plates with general structural stress distributions

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Abstract

Complex stress distributions often exist in ocean engineering structures. This stress influences structural vibrations. Finite Element Methods exhibit some shortcomings for solving non-uniform stress problems, such as an unclear physical interpretation, complicated operation, and large number of computations. Analytical methods research considers mainly uniform stress problems, and often, their methods cannot be applied in practical marine structures with non-uniform stress. In this paper, an analytical method is proposed to solve the vibration of plates with general stress distributions. Non-uniform stress is expressed as a special series, and the stress influence is inserted into a vibration equation that is solved through decoupling to obtain an analytical solution. This method has been verified using numerical examples and can be used in arbitrary stress distribution cases. This method requires fewer computations and it provides a clearer physical interpretation, so it has advantages in some qualitative research.

Keywords: Structural stress; Plate structure; Vibration; Analytical method

1. Introduction

The complexity and special working environment of some practical ocean engineering structures means that often, stress exists in the structure prior to it being subjected to a work load. These stresses include artificial stresses, such as prestress in structural connections; stress caused by manufacturing, such as welding residual and assembly stresses; and stress caused by complex and special working environments, such as submarine shell stresses caused by hydrostatic pressure. It is worth noting that these kinds of stresses could change structural performance.

A large amount of research effort has been devoted to studying the influence of stress on structural strength and fatigue (Dong, 2001; Gannon, 2012; Khan, 2011; Niemi, 1995; Paik, 2012). As we know, the structural vibration is always a research hotspot (Cho, 2016; Senjanović, 2015, 2016). However, in comparison, the influence of stress on vibration is often ignored and limited related research exists in this area. Doong (1987) applied high-order shear deformation theory to derive the initial stress thick plate vibration control equation, and compared results with reference data (Brunelle and Robertson, 1976). Fuller and Fahy (1982) considered pressure caused by fluid filling a cylinder and derived its free vibration equation. Liu and Zhang used the wave propagation method to study the effect of hydrostatic pressure fields on

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vibration characteristics (Liu, 2010, 2011; Zhang, 2001, 2002). However, they focused mainly on uniform distributed stress. The influence of stress concentration around holes on vibration has also been studied (Yahnioglu, 2007). Gao (2002, 2014) compared experimentally a plate with and without welding stress. Although their research focused on non-uniform distributed stress, it is limited to specified stress-distribution types and cannot be applied to other distributions. Non-uniform stress distributions are often encountered in ocean engineering structures and these distributions vary in different working environments. The former methods cannot be used to solve this practical stress problem, and a method is required that can solve structural vibrations with general stress distributions.

Although the Finite Element Method (FEM) can be used to investigate structural dynamics with non-uniform stress distributions (Chen et al., 2014), it cannot explain the essential relationship between structural stress and vibration. Complicated operations are required to exert a non-uniform stress distribution and reruns are required when the stress distribution changes. FEM requires high modelling costs, especially for large-scale structures. So, it is inconvenient to conduct research on the stress-caused influence on vibration by FEM. Therefore, it is necessary to develop alternative analytical methods.

This work aims to provide a unified and efficient solution for vibration analysis of structures with general stress distributions. Plate structures are discussed and an analytical method is proposed to solve the vibration problem of non-uniform stressed plates. Using the proposed method, structural stress, regardless of its distribution and value, is expressed as a special series that can express almost all of the stress distributions and achieve partial decoupling among structural modes in the final vibration equation. The analytical solution is obtained by solving this decoupling equation. Finally, this method is verified using numerical examples. The analytical solution can be applied to a structure with arbitrary distributed stress, so it has a wider range of applications than previous analytical methods. It also requires less computation and provides a clearer physical interpretation than FEM, so it is more suited for qualitative research on the relationship between stress and vibration. The proposed method can be applied in the vibration analysis of ships and offshore structures with a non-uniform distributed stress, such as submarine welding stresses, which are distributed only near the joints; risers' varying stresses that are caused by hydrostatic pressure at different water depths; and varying seabed pipeline stresses that are caused by in-pipe fluid with different fluid velocities and temperatures.

Structural stress distributions can be obtained by using some mature methods according to different stress factors, for example, the welding stress (Li, 2010; Radaj, 2012), the hydrostatic pressure-induced stress (Cao, 1989; Liu, 2009), and the in-pipe fluid-induced stress (Bokaian, 2004; Zhang, 2001). Once the stress distribution has been determined, structural vibration with a specific distributed stress can be obtained rapidly using the presented method. This stress-considering vibration result is closer to the reality. By comparing the non-stressed structure's vibration, the level and feature of the stress influence on vibration can be analysed. Based on the vibration results and stress influences, further steps can be taken to resist vibration.

The proposed method also has application in many other engineering applications, such as vibration optimization and damage identification. In future, it may be possible to combine the presented method with innovative materials, such as Functionally Graded Materials (FGMs) (Belabed, 2014; Bellifa, 2016; Bennoun, 2016; Bouderba, 2013; Bourada, 2015; Hamidi, 2015; Hebali, 2014; Mahi, 2015; Meziane, 2014; Tounsi, 2013; Ait Yahia et al., 2015; Zidi, 2014) FGMs are heterogeneous materials in which the material properties are varied continuously from point to point. At each interface, the material is chosen according to specific applications and environmental loadings. Nowadays, FGM is being used increasingly in engineering.

The remainder of this paper is organised as follows: Section 2 builds the vibration equation of the structure with general stress distributions. Section 3 presents the solution procedures of the equation derived in Section 2. In Section 4, some numerical examples are used to verify this method. Section 5 presents the conclusions.

2. Theoretical formulations

2.1. Description of the model

In most previous methods, the stress value is seen as an invariant because it considers mainly the uniform stress distribution case (Fuller and Fahy, 1982; Liu et al., 2010, 2011; Zhang et al., 2001a,b, 2002), and these invariants are substituted into the vibration equation to obtain the vibration equation of uniform stressed structures. However, the aim of this study is a plate with arbitrary stress distributions as shown in Fig. 1, and the stress value varies with different locations. So, the basic vibration equation of structure with a non-uniform stress needs to be derived. In Fig. 1, h is the thickness; a is the plate length; b is the plate width; the x axis represents the length direction; the y axis represents the width direction; and u, v, and w are the neutral plane displacements in the x, y, and z directions, respectively. Stresses can be divided into a normal stress σ_x^r and σ_y^r and a shear stress τ_{xy}^r, and their stress amplitudes are functions of x and y.

Before the derivation, the following assumptions are made:

(1) Fluid−Structural Interaction is not considered in this paper.
(2) Structural stresses and stresses caused by vibration satisfy the linear superposition.
(3) The vibration satisfies the small elastic deformation condition.
(4) The stress is distributed uniformly in the thickness direction.
(5) Stress relief is not considered in this paper.

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