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# Particle size effects on soil reflectance explained by an analytical radiative transfer model

Morteza Sadeghi[a,*], Ebrahim Babaeian[b], Markus Tuller[b], Scott B. Jones[a]

[a] Department of Plants, Soils and Climate, Utah State University, Logan, UT, United States
[b] Department of Soil, Water and Environmental Science, The University of Arizona, Tucson, AZ, United States

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ABSTRACT

Experimental evidence points to an intimate link between soil reflectance, *R*, and particle/aggregate diameter, *D*. Based on this strong correlation, various statistical methods for remote and proximal sensing of soil texture and hydraulic properties have been developed. In this paper, we derive a more fundamental and physically-based analytical radiative transfer model that yields a closed-form functional *R*(*D*) relationship for dry soils. Despite several simplifying assumptions, the proposed model shows good agreement with measured spectral reflectance (350–2500 nm) data of six soils covering a broad range of textures, colors, and mineralogies. The proposed S-shaped *R*(*D*) function resembles cumulative particle and pore size distributions as well as the soil water characteristic function. These analogies may potentially lead to new avenues for developing novel physical models for extracting important soil properties from remotely sensed reflectance data.

## 1. Introduction

An abiding goal of terrestrial remote sensing is to extract land surface properties from remote observations of spectral reflectance. Reflectance spectroscopy in the optical range (i.e., 350–2500 nm) has been widely applied as an efficient tool for quantifying soil constituents including minerals (Clark et al., 1990; Kruse et al., 2003; Mulder et al., 2013; Omran, 2017), organic matter (Ingleby and Crowe, 1999; Daniel et al., 2004; He et al., 2009; Leue et al., 2017), and water (Lobell and Asner, 2002; Whiting et al., 2004; Sadeghi et al., 2015; Zeng et al., 2016). Soil reflectance spectra are also affected by particle size (Bowers and Hanks, 1965; Hunt and Vincent, 1968; Bänninger et al., 2006), bulk density (Dematte et al., 2010; Bachmann et al., 2014; Carson et al., 2015; Tian and Philpot, 2017), and surface roughness (Cierniewski, 1987; Cierniewski et al., 2004; Wu et al., 2009; Piekarczyk et al., 2016).

An intimate relationship between reflectance of particulate materials (e.g., aerosols, solid powders, snow, etc.) and particle size exists (Hapke, 1981; Sun and Zhao, 2011; Myers et al., 2015). For soils, this relationship has been the basis for the development of various methods for remote and proximal sensing of basic properties. Recent work of Liao et al. (2013) and Lakshmi et al. (2015) on soil texture, Hermansen et al. (2017) on particle size distribution, Böttcher et al. (2012) on soil surface macroporosity, and Babaeian et al. (2015a, 2015b) on soil hydraulic properties are only a few examples, where mostly data-driven empirical relationships have been established between the remotely sensed signal and target variables. To more fundamentally understand these relationships, we strived to derive a physically based radiative transfer model relating soil reflectance, *R*, to particle diameter, *D*.

Theoretical work on particle size effects is abundant in atmospheric sciences, particularly on radiation scattering by aerosols (Yamamoto and Tanaka, 1969; Grassl, 1971; Shaw, 1979; He et al., 2014; Zhao et al., 2014; Zhao and Li, 2015). Although the same laws of radiative transfer may be potentially applicable for soils, theoretical models explaining the *R*(*D*) functional relationship for soils are rare. Most studies focused on the *R*(*D*) relationship of soils are experimental or empirical in nature, showing the expected inverse correlation of soil reflectance and particle size (Bowers and Hanks, 1965; Hunt and Vincent, 1968; Sinha, 1987; Cooper and Mustard, 1999; Okin and Painter, 2004; Sun et al., 2014). Studies of Bänninger et al. (2006) and Leue et al. (2010) are among the very few that theoretically show the effects of particle size on soil reflectance based on the beam-tracing model (BTM) of Bänninger et al. (2006). The BTM numerically calculates the optical path of a number of incident radiation beams within and between soil particles, and this way, can simulate soil reflectance and transmittance. This computer model, however, does not provide an analytical relationship for *R*(*D*).

In this paper we introduce a simple, physically-based and analytical radiative transfer model to obtain a closed-form functional relationship for *R*(*D*). Model performance is evaluated with measured spectral reflectance data for different soils of various particle/aggregate sizes.

* Corresponding author at: 4820 Old Main Hill, Logan, UT 84322-4820, United States.
E-mail addresses: morteza.sadeghi@usu.edu (M. Sadeghi), ebabaeian@email.arizona.edu (E. Babaeian), mtuller@email.arizona.edu (M. Tuller), scott.jones@usu.edu (S.B. Jones).

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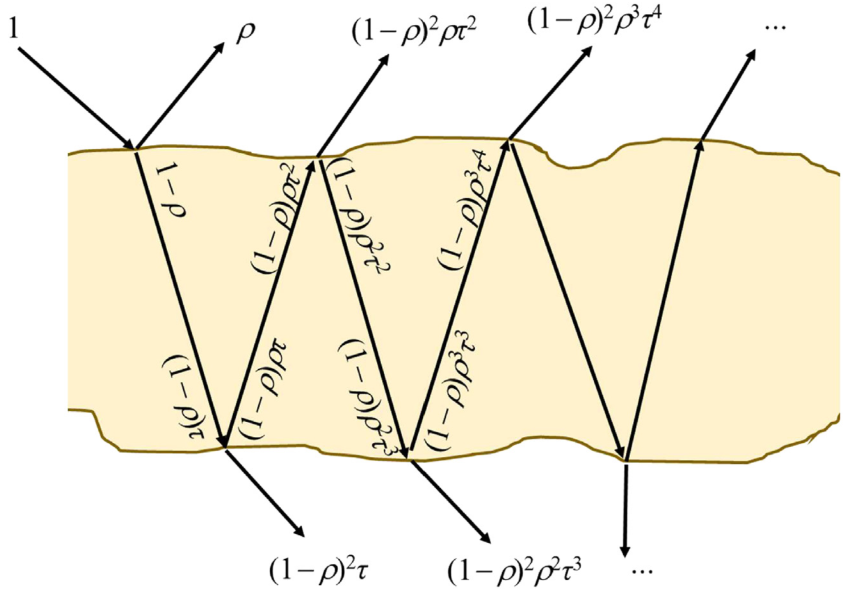


**Fig. 1.** Sketch illustrating the surface reflectance model; ρ is the reflectivity of the soil-air interface and $\tau$ is the transmissivity of the surface soil layer.

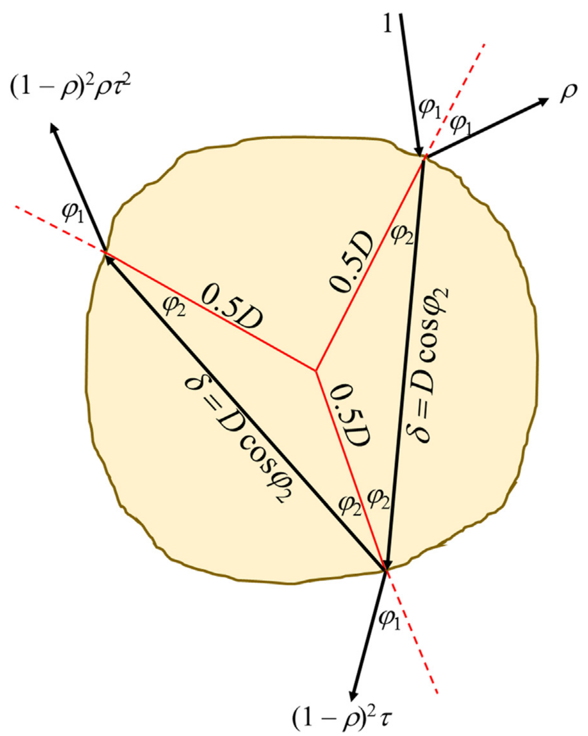


**Fig. 2.** Sketch illustrating the path length of a random radiation beam passing through a nearly spherical soil particle.

## 2. Theoretical approach

We first postulate that soil particles or aggregates have a narrow size distribution, such that an average (effective) particle/aggregate size can be assumed. Then, we model reflectance, $R$, as a function of the average particle/aggregate diameter, $D$. Although this postulation may not always reflect natural conditions, it is made here to clearly demonstrate particle/aggregate size effects on soil reflectance. For simplicity, hereinafter we refer to both particles and aggregates as "particles" and assume that the same models hold for both.

Soil surface reflectance, $R_{surface}$, (i.e., fraction of light reflected directly from surface particles) and volume reflectance, $R_{volume}$, (i.e., fraction of light reflected due to scattering by underlying particles) are individually calculated based on simple beam-tracing models with both contributing to the total reflectance, $R$:

$$R = R_{surface} + R_{volume} \tag{1}$$

For surface reflectance, we assume that the surface particles form a thin tortuous layer such as depicted in Fig. 1. The solution to this problem is approximated with Stokes' (1860) radiative transfer model for a slab considering infinite reflections from and transmissions through the upper and lower interfaces (Fig. 1). Surface reflectance is accordingly calculated as:

$$R_{surface} = \rho + (1-\rho)^2\rho\tau^2 + (1-\rho)^2\rho^3\tau^4 + (1-\rho)^2\rho^5\tau^6 + \ldots$$
$$= \rho + [(1-\rho)^2\rho\tau^2] \times \sum_{i=0}^{\infty} (\rho^2\tau^2)^i \tag{2}$$

where ρ is dimensionless reflectivity of the soil-air interface and $\tau$ is dimensionless transmissivity of the surface soil layer (i.e., due to layer absorption the light flux will be reduced by a fraction of $1 - \tau$ when passing through the layer). For the sake of simplicity, we assume that the reflectivity of the soil-air interface is the same for incoming and outgoing radiation beams. This assumption was introduced by Stokes (1860), however, others considered different reflectivity values for incoming and outgoing radiation beams (e.g., see Fig. 5.15 in Hapke, 2012).

According to the geometric series, $\sum_{i=0}^{\infty} x^i = (1-x)^{-1}$, the surface reflectance in Eq. (2) can be written in closed form as:

$$R_{surface} = \rho + \frac{(1-\rho)^2\rho\tau^2}{1-\rho^2\tau^2} \tag{3}$$

The transmissivity $\tau$ of the layer is related to its absorption coefficient through the Beer-Lambert law:

$$\tau = \exp(-k\delta) \tag{4}$$