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Theoretical bounds for the exponent in the empirical power-law advance-time curve for surface flow

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ABSTRACT

A fundamental and widely applied concept used to study surface flow processes is the advance-time curve, which indicates the time at which water arrives at any given distance along a field length. The advance-time curve is typically characterized by an empirical power law with an exponent r and a numerical prefactor p (i.e., $x = pt^r$). In the literature, different values of r have been reported for various situations and types of surface irrigation. Invoking concepts from percolation theory, a theoretical approach from statistical physics for quantifying transport properties in complex systems, we relate the exponent r to the backbone fractal dimension D_b ($r = 1/D_b$). The backbone, through which transport occurs predominantly in a system, is formed by multiply connected loops composed of several paths that bring fluid from an initial point to a final point. The backbone fractal dimension D_b, characterizing the complex structure of the backbone, depends on two factors: dimensionality of the system (e.g., two or three dimensions) and percolation class (e.g., random or invasion percolation with/without trapping). We show that the theoretical bounds of D_b are well in agreement with experimental ranges of r reported in the literature for two furrow and border irrigation systems. We further use the value of D_b from the optimal path class of percolation theory to estimate the advance-time curves of four furrows and seven irrigation cycles (i.e., 28 experiments). The optimal path is the most energetically favorable path through a system. Excellent agreement was obtained between the estimated and observed curves. We also discuss the indirect effects of initial water content, inflow rate, surface slope, and infiltration rate (soil texture) on system dimensionality, percolation class and, consequently, the backbone fractal dimension value. More specifically, we postulate that for closely-spaced furrows with steep slopes, low infiltration rates, and relatively high inflow rates, the wetting front advance will be mostly quasi one-dimensional. Since the backbone fractal dimension in one dimension is 1, one should expect the exponent r to be near 1.

1. Introduction

Modeling water flow and solute transport in surface irrigation systems requires knowledge of how quickly water moves over the soil-air interface along a furrow or border. Analyses of this type lead to the advance time curve, which describes the balance among water that already exists in a soil (referred to as the initial water content), water that infiltrates into the soil, and water that traverses on the surface. One of the most widely used forms in the literature for the advance time curve, which indicates the time at which water arrives at a given distance in a field, is the following empirical power law

$$x = pt^r \tag{1}$$

in which t is the time needed for the wetting front to reach distance x, p is a numerical prefactor and r is an empirical exponent. The advance-time curve is a function of various factors, such as initial watercontent, soil texture, inflow rate, and surface slope. The effects of such factors are lumped into the parameters p and r in Eq. (1). However, since Eq. (1) is based mostly on empiricism, it is not clear how p and r vary from one field or irrigation cycle to another.

Different values of r have been reported experimentally. For example, Elliott and Walker (1982) estimated the parameters p and r by directly fitting Eq. (1) to two measured data points of the advance-time

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curve. Such a procedure, known as the two-point method, was first used for furrow irrigation and later applied to basin and border irrigation systems. In basin irrigation, probably the most common form of surface irrigation, a field is level in all directions and encompassed by a ditch to avoid runoff. Basins are commonly square or rectangular in shape. However, they may exist in all sorts of irregular configurations. Border irrigation is a more general type compared to basin irrigation. Borders are typically long rectangular in shape, with free draining conditions at the lower end. In furrow irrigation, the field surface is channeled along its primary direction using furrows. The main difference between furrow and border/basin irrigation is that for the former flow into each furrow is independently set and controlled, while for the latter flow is controlled on a border by border or basin by basin basis.

Shepard et al. (1993) fixed r at 0.5 and optimized p by using typically the last measured point on the advance-time curve. Their method is known as the one-point method. Valiantzas et al. (2001) later presented another one-point method. Although they did not fix the value of r, their method requires a trial and error procedure to determine r and p from the last point of the advance-time curve.

In addition to one- and two-point methods, the values of p and r can be determined by directly fitting Eq. (1) to advance-time data if measurements are available. For instance, Serralheiro (1995) estimated r for a Mediterranean soil (southern Portugal) using furrows of different slopes, and found $0.52 \le r \le 0.94$ (see their Table 1). In another study, Alvarez (2003) reported $0.58 \le r \le 0.72$ for different types of soils (such as Gleysols, Vertisols, Acrisols, and Ferrasols) for furrows of lengths between 240 and 380 m, with Manning's resistance coefficient n ranging from 0.02 to 0.04. The Manning equation is an empirical formula for estimating the cross-sectional average velocity (v) of water flowing along an open channel from the hydraulic radius R_h, the surface slope S, and Manning's resistance coefficient n (i.e., $v = R_h^{2/3}S^{1/2}/n$). For the estimation of Manning's resistance coefficient for flow in a field, see Harun-Ur-Rashid (1990) and Li and Zhang (2001).

In Table 1, we summarize the range of r value reported in different studies under various conditions. Experimental evidence indicates clearly that r is a function of several factors, such as soil type, initial water content, soil surface slope, and type of surface irrigation. Thus, one should expect r to vary from one soil and/or an irrigation cycle to another.

The main objective of this study was to invoke concepts from percolation theory, a theoretical approach from statistical physics for quantifying transport properties in complex systems, to shed light on the value of r, and relate this parameter to the backbone fractal dimension D_b that characterizes the backbone structure for percolation. In what follows, we first introduce percolation theory and present its fundamental concepts. We then propose a power law from percolation theory and show the link between the backbone fractal dimension D_b and the advance-time curve exponent r in Eq. (1). We next compare our theoretical results with experiments for furrow and border irrigation systems.

2. Percolation theory

Percolation theory, introduced in its present form by Broadbent and Hammersley (1957), provides a promising theoretical framework from statistical physics to study interconnectivity and its effects on transport properties in heterogeneous systems and networks, such as soils. Broadbent and Hammersley (1957) studied plant disease spreading in an orchard whose trees were located on the intersections of a square lattice. As expected, when the distance between aligned trees increases, the probability of spreading a disease decreases. Eventually the distance between trees would reach a critical value above which the disease cannot spread over the orchard (Feder, 1988).

Percolation theory exists in three main forms: bond, site, and continuum percolation. In bond percolation, bonds are represented by randomly distributed segments that are open to flow and transport (see Fig. 1a). They act as microscopic conducting elements. In site percolation, sites are the line intersections and bonds are the segments

Table 1
Values of the exponent r in Eq. (1) reported in the literature for various conditions.

Reference	Irrigation system	No. of samples	r	Ave. r	Remarks
Alvarez (2003)	Furrow	12	0.58–0.72	0.66	From various references, see their Table 1
Abbasi et al. (2003)	Furrow	3	0.62–0.91	0.76	Maricopa Agricultural Center, Phoenix AZ
Khatri and Smith (2006)	Furrow	27	0.73–0.97	0.86	Cotton field T, Southern Queensland
	Furrow	17	0.62–0.85	0.72	Cotton field C, Southern Queensland
Bautista et al. (2009)	Furrow	4	0.57–0.81	0.67	Four furrows of various lengths and soil textures
Abbasi et al. (2009a)	Furrow	3	0.69–0.91	0.77	Seed & Plant Improvement Res. Inst., Karaj, Iran
Abbasi et al. (2009b)	Furrow	6	0.62–0.91	0.76	Research Station for Tobacco, Urmia, Iran
Ebrahimian et al. (2010)	Furrow	5	0.52-0.77	0.63	From various references, see their Table 1
	Border	6	0.48–0.68	0.58	From various references, see their Table 2
Ebrahimian (2014)	CFI[a]	7	0.65–0.86	0.76	Maize field, Karaj, Iran
	FFI	7	0.65–0.70	0.68	Maize field, Karaj, Iran
	AFI	7	0.63–0.75	0.69	Maize field, Karaj, Iran

[a] CFI: conventional furrow irrigation, FFI: fixed alternate furrow irrigation, AFI: variable alternate furrow irrigation.

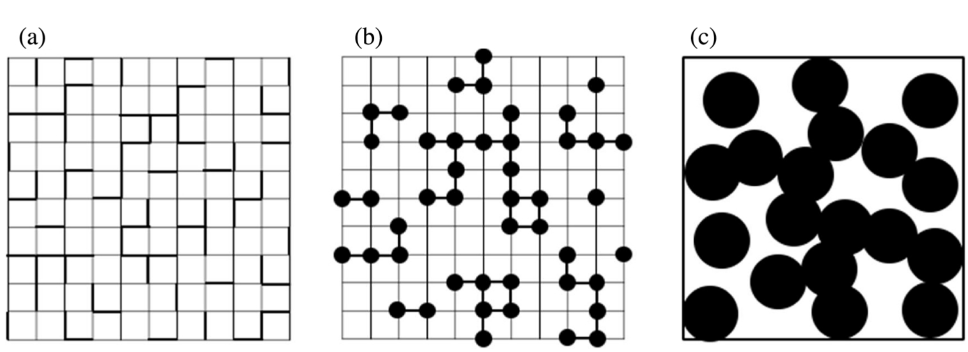

Fig. 1. Schematic (a) bond, (b) site, and (c) continuum percolation forms on a two-dimensional lattice. In bond percolation, segments are randomly distributed within the lattice, while in site percolation sites are the line intersections, randomly selected, and bonds are the segments connecting sites. In continuum percolation, circular voids are randomly placed within the lattice.