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FLFP: A fuzzy linear fractional programming approach with double-sided fuzziness for optimal irrigation water allocation

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ABSTRACT

In this study, a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness is developed for optimal irrigation water allocation under uncertainty. The FLFP model can be derived from incorporating double-sided fuzzy chance-constrained programming (DFCCP) into linear fractional programming (LFP) optimization framework. The developed model can deal with uncertainty presented as fuzziness in both right-hand and left-hand sides of constraints. Moreover, it has advantages in: (1) addressing two objectives directly without considering subjective factors, (2) effectively reflecting economic water productivity between total system economic benefit and total irrigation water use, (3) introducing the concept of confidence levels of fuzzy constraints-satisfaction under both the minimum and maximum reliabilities to generate more flexible solutions and (4) facilitating in-depth analysis of interrelationships among economic water productivity, system benefits and varying confidence levels. The model is applied to a case study of irrigation water allocation in the middle reaches of Heihe River Basin, northwest China. The optimal irrigation water allocation solutions from the FLFP model can be obtained. These results can provide decision-support when deciding on selecting reasonable irrigation water resources management and agricultural production.

1. Introduction

Nowadays, it is commonly believed that growing population and diminishing water availability in China are jointly affect the contradiction between water supply and water demand, especially in arid or semiarid regions dominated by agriculture. Moreover, irrigated agriculture is the biggest consumer attributing to water use, consuming 70% of total water use in the world (Kang et al., 2017). Irrigation water management naturally becomes worldwide concerns for agricultural production and livelihood security with limited water resources (Storm et al., 2011; Parsinejad et al., 2013). Therefore, how to improve agricultural water use efficiency or agricultural water productivity and further effectively allocate irrigation water resources is expected to ensure food security and promote sustainable development of agriculture.

To address the above concerns, many optimization methods or models have been developed for optimal irrigation water allocation (Dudley et al., 1971; Rao et al., 1990; Mainuddin et al., 1997; Shangguan et al., 2002; Noory et al., 2012; Yang et al., 2015; Zhang et al., 2017a). However, among these studies, the objectives of model almost focus on the system benefits and few studies optimize the agricultural water use efficiency. Agricultural water use efficiency is defined as the production per unit of irrigation application (Playán and Mateos, 2006), which is also known as agricultural water productivity reflecting the system efficiency. More importantly, in many developing countries, irrigation water management suffers from lower water use efficiency and the irrigation water resources are not utilized efficiently (Li et al., 2016a). In water limited areas where water is the most limiting factor, to obtain higher water productivity may be economically more profitable for the farmer and it is more likely to be considered rather than higher crop yield or economic benefits. This problem can be quantitatively described by an optimization model using linear fractional programming (LFP). The LFP model is capable of facilitating the analysis of system efficiency through transforming a bi-objective (e.g. cost and volume, output and input) problem into a ratio one, especially for addressing the issue with better achievements per unit of inputs (Lara and Stancu-Minasian, 1999; Gomez et al., 2006; Guo et al., 2014; Zhang and Guo, 2017a). However, the LFP model has difficulty in tackling uncertainties in irrigation water allocation problems.

Uncertainties existing in irrigation water management systems should be considered strategically. Generally, the irrigation

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water management basically consists of various parameters, such as hydrological elements, economic parameters and other related parameters, which are not easily quantified and defined in the model (Regulwar and Gurav 2011; Elliott et al., 2014; Zhang et al., 2017b). There are many inexact optimization methods mainly including stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP) and interval linear programming (ILP) were developed to tackle uncertainties (Huang, 1996; Shastri and Diwekar, 2006; Kerachian and Karamouz, 2007; Guo et al., 2010; Li et al., 2013; Liu et al., 2014; Li et al., 2015; Nematian, 2016; Wang et al., 2016). In detail, SMP is generally used to deal with random uncertainties (i.e. hydrological parameters) in the irrigation water allocation problems. However, the data requirement to specify parameter with probability distribution functions (PDFs) and intensive computational burden may result in difficulties in its practical applications (Li et al., 2016a). ILP can deal with all uncertain parameters expressed as interval values without considering distribution information. However, it is incapable of tackling problems with violation of system constraints (Huang, 1996). In practical, some parameters exhibit features of vagueness and imprecision, they are estimated empirically and subject to human judgments, and can better be expressed by fuzzy membership functions (Xu and Qin, 2010). Moreover, FMP has a lower data requirements but reflects more flexible information in practical application because the related membership functions are more easily defined than PDFs (Xu et al., 2010; Wan and Li, 2015). Besides, when the violation of system constraints exists in the optimization model, the chance-constrained programming with fuzzy parameters needs to be introduced into the general optimization framework. Therefore, chance-constrained programming (CCP) was extended from stochastic (PDFs) to fuzzy (possibility distribution) environments (Liu and Iwamura, 1998). Similar to chance-constrained programming with random uncertainties models, fuzzy chance-constrained programming (FCCP) also requires that the fuzzy constraints be transformed into deterministic ones at predetermined confidence levels, which has a lower computational burden and generates more flexible solutions. Therefore, FCCP is an improved FMP method and it's effective for tackling fuzzy uncertainties and violation of system constraints (Xu and Qin, 2010). However, in many practical applications, fuzzy uncertainties may exist in both sides of constraints within the model. Thus, double-sided FCCP (DFCCP) method is a better choice for addressing these complexities. Nevertheless, DFCCP method cannot deal with ratio optimization problems. In addition, there are few studies on DFCCP method for irrigation water allocation under uncertainty.

Therefore, this study aims to develop a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness for optimal irrigation water allocation under uncertainty. Technique of DFCCP will be incorporated into LFP optimization framework. Therefore, the developed model can deal with uncertainty presented as fuzziness in both left-hand side and right-hand side components of constraints. Moreover, it has advantages in: (1) addressing two objectives simultaneously without considering subjective factors, (2) effectively reflecting economic water productivity, (3) introducing the concept of confidence levels of fuzzy constraints-satisfaction under both the minimum and maximum reliabilities to generate more flexible solutions and (4) facilitating in-depth analysis of interrelationships among economic water productivity, system benefits and varying confidence levels. The model is applied to a case study to demonstrate its applicability. The study area is a major agricultural production base in China while it currently faces increasingly severe water shortages problems. The objective is to maximize the economic water productivity (i.e. to obtain maximum system benefits with minimum irrigation water allocation) to allocate the limited irrigation water to different crops and different subareas. The optimal solutions of irrigation water allocation can be generated under each confidence level, which can be used to facilitate the analysis for the various irrigation water management schemes under complex uncertainties. The framework of this study is illustrated in (Fig. 1).

2. Formulation of the FLFP approach

Consider an agricultural irrigation system in a region where multiple crops need to be irrigated in different irrigation districts. The managers are responsible for making irrigation plans for these competing water users. Generally, a higher agricultural water productivity is a greater concern under limited water conditions for creating a sustainable agriculture system. Such a problem can be effectively addressed by linear fractional programming (LFP) model (Charnes and Cooper, 1963; Zhu and Huang, 2011). Moreover, when both left-hand and right-hand side parameters in the constraints are of fuzzy features and can be presents as possibility distributions, and the violation of system constraints exists in the optimization model, thus the double-sided fuzzy chance-constrained programming (DFCCP) method can be adopted (Fiedler et al., 2006; Xu et al., 2012). For example, in this study, parameters associated with the water availability and the rate of water loss during water conveyance are expressed as fuzzy sets. Therefore, in response to above concerns, a fuzzy linear fractional programming (FLFP) approach with double-sided fuzziness is developed by integrating the DFCCP method into LFP framework. Consequently, based on FLFP approach, the study problem for irrigation water allocation to different crops in different subarea can be formulated as follows:

System objective:

$$\max f = \frac{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(NB_{ij} - CP_{ij}\right) A_{ij}\left[a_j\left(SW_{ij} + GW_{ij} + P_{e,i}\right) + b_j\right] - \sum_{i=1}^{12}\sum_{j=1}^{3} A_{ij}\left[CS_i SW_{ij}/\eta_s + CG_i GW_{ij}/\eta_g\right]}{\sum_{i=1}^{12}\sum_{j=1}^{3}\left(SW_{ij}/\eta_s + GW_{ij}/\eta_g\right)} \tag{1a}$$

where f is the objective function reflecting economic water productivity (10^6 Yuan/m^3); i denotes irrigation district (i = 1, 2, ⋯, 12); j denotes type of crop (j = 1, 2, 3); NB_{ij} is the price of crop j in irrigation district i (Yuan/kg); CP_{ij} is the cost of crop production of crop j in irrigation district i, which includes all other costs such as seed, fertilizer, pesticides, machinery, harvesting, marketing, drying and unexpected costs. This cost is independent of the amount of applied irrigation water (Yuan/kg); A_{ij} is the irrigated area of crop j in irrigation district i (ha); a_j and b_j are the coefficients of linear crop water production function of crop j; SW_{ij} and GW_{ij} are the decision variables, which means that the allocated surface water and groundwater (m^3/ha); $P_{e,i}$ is the effective precipitation of irrigation district i (m^3/ha); CS_i, CG_i are the cost of surface water and groundwater use in irrigation district i (Yuan/m^3); η_s and η_g are the irrigation water use coefficients of surface water and ground water respectively.

Constraints

The objective is subjected to a series of constraints in water availability, irrigation water demand, fairness, and other related concerns. These constraints can determine the reasonable decision space of decision variables.