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Research papers

Hydro-instability characteristics of Bingham fluid flow as in the Yellow River

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A R T I C L E I N F O

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A B S T R A C T

Hydro-instability of hyper-concentrated flow in the Yellow River is one of the focuses of Chinese sediment research. In this research, the constitutive relations and the equations of the hydro-instability are first explores. Based on the experimental results and field data, Bingham fluid model is simple and appropriate for the hyper-concentrated flow. Next, the hydro-instability characteristics of hyper-concentrated flow in the Yellow River are discussed, which is judged with the neutral curve by using perturbation method and the summarized Bingham fluid model; then, the variation of the neutral curve is analyzed with sediment parameters, which are size distribution and its median, representing as critical concentration and concentration. Additionally, the variations of ratio of such characteristic parameters as Reynolds number of turbid water to clear water are obtained to represent the relation between Bingham fluid and ordinary Newtonian fluid, which can be applied to judge the flow pattern in the Yellow River.

1. Introduction

Yellow River, the second longest river in China, is famous for its high sediment concentration. The time-averaged sediment concentration is about 35 kg/m^3, which is times of ordinary rivers, over 100 times of the Amazon River and even more than 10 times of the Indus River. The sediment concentration can be 300 kg/m^{3}in the Yellow River during floods. Moreover, the sediment concentration in the middle reach can be as high as 1500 kg/m^3, with a peak value over 1600 kg/m^3 (Wan and Wang, 1994).

The most part of sediment in the Yellow River is suspended load. According to measurements, the annual weight of sediment carried by flows from the upper and middle reaches to the lower reach before the construction of large reservoirs (1919–1960) is 1600 million tons, of which only 400 million tons of sediment deposit on riverbed and 1200 million tons of sediment flow to the sea as suspended load (Wang, 2002).

The flow in the Yellow River often shows specific phenomena due to the high concentration of sediment suspended in water. During the dry season, the free surface of the flow is as peaceful as a mirror, and the center of the river even has a "plug" form, which means there is no relative displacement between layers; while, during the flood season, the free surface and velocity of the flow fluctuates significantly with the increase of timely-averaged velocity and water depth. This results in great fluctuations of weight of sediment carried by flow, causing sudden erosions or depositions on the riverbed of the Yellow River (Chien and Wan, 1998).

These phenomena are closely related with the hydro-instability characteristics of hyper-concentrated flow, especially in the Yellow River. The Reynolds number of the flow in an ordinary river is always greater than critical Reynolds number. So, the flow is always turbulent. However, the situations become different for the flow in a river with a high concentration of sediment. When the sediment concentration exceeds a certain value, the viscosity of the fluid would increase rapidly, causing Reynolds number of the flow to drop dramatically to a value smaller than the critical Reynolds number. Then the flow, which is turbulent under normal conditions, becomes a homogeneous laminar flow during the dry season. The free surface appears like a mirror, and the center of the river flows like a "plug" form. While during the flood season, the Reynolds number of the flow increases accordingly with the increase of averaged velocity and water depth. When the Reynolds number exceeds the critical Reynolds number, the flow becomes turbulent, and the flow reveals itself with typical turbulence characteristics. The instantaneous velocity in the river varies, and the free surface fluctuates with time. The carrying capacity of the flow changes accordingly, resulting in fluctuations in weight of sediment carried by water, which leads to the sudden erosion or deposition in a local position of the river bed.

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In order to find out the reasons of the specific phenomena in the Yellow River, the investigation on the hydro-instability characteristics of the hyper-concentrated flow was conducted as the key of this research.

2. Review

The basis to study stability characteristics of the hyper-concentrated flow is to analyze the flow properties of the Yellow River. Many investigators have already described the relation between them and conducted related physical experiments. Sha (1961) stated that the rheology of the flow with low and middle concentrations of sediment is closely related to sediment concentration and median diameter; he then carried out an equivalent-viscosity model for engineering practice. Later, Bai and Xu (2010) applied Sha (1961) model to their calculation on the flow with middle concentration of sediment. However, many researchers did not think the model is appropriate for hyper-concentration flow. One of the most important reasons is the appearing of the "plug" form in the center of the Yellow River, which cannot be explained with any Newtonian models or equivalent-viscosity models (Chien and Wan, 1998; Fei, 1994). Julien and Lan (1991) as well as O'Brien and Julien (1988) stated that the Bingham plastic model could be applied to describe the rheological characteristics of such hyper-concentrated fluids as mine tailings, sewage sludge, mud, muddy debris and so on. The reason is that shear stress in this model linearly increases with shear rate only when it exceeds the yield stress. And the existence of the yield stress is one of the key factors causing the "plug" form. Additionally, the results of the experiments on the flow properties of the Yellow River (Wang and Qian, 1984) show that the flow with a large amount of cohesive sediment transforms from Newtonian flow to Bingham Plastic flow, and the value of yield stress is determined by the proportion of fine particles. Furthermore, their assumptions have already been proved by the experimental data (Fei, 1983; Qian, 1989; Fei, 1991; Fei, 1994) of the flow in the Yellow River.

Despite the significant progress in understanding the properties of the hyper-concentrated flow in the Yellow River, there are only a few studies discussing the mechanisms and characteristics of the hyper-concentrated flow of the Yellow River. For example, Engelund and Wan (1984) stated that the characteristics of flow with such high concentration sediment as the flow in the Yellow River are associated with the relation between its slope and discharge. They then established a discharge model, and analyzed the different influences of two kinds of materials on the characteristics of the flow. Wanget al. (1990) and Wang (2002) introduced the Bingham plastic model to one-dimensional Saint-Venant equation, and found that the yield stress of the fluid leads to the generating and developing of the unstable flow, while the viscosity is an inhibitory factor for the flow. Therefore, the stability of the hyper-concentrated flow depends on the competition and compromise of these two parameters. Shu and Zhou (2006) derived an approximate expression of conjugate depths during laminar flow of a non-Newtonian Bingham fluid over a horizontal plane. Similar work also includes the researches done by Haldenwang and Slatter (2006), Ugarelli and Federico (2007) and so on.

Based on the discussion above, almost all of the previous models only consider Froude number rather than Reynolds number, such as those established by Wanget al. (1990), Wang (2002), Shu and Zhou (2006), Ugarelli and Federico (2007) and so on. However, Reynolds number is a very important factor in the study of the stability characteristics of hyper-concentrated flow. This is because: (1) Reynolds number is the ratio of inertia force to viscosity. Additionally, viscosity is the key factor leading to the incipient motion of particles. (2) Reynolds number is the key parameter that judges the transition from laminar flow to turbulence, which dramatically changes the flow structure. With energy transferred from flow to sediment, this transition further causes the variation in the weight of sediment carried by the flow, i.e. carrying capacity of the flow.

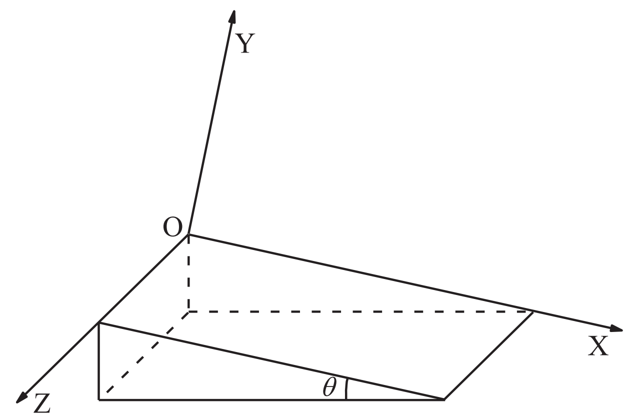

Fig. 1. Schematic diagram of coordinate system.

This research aims to study the hydrodynamic instability characteristics of hyper-concentrated flows. The flow in the Yellow River was taken as an example and Reynolds number was considered as one of the key factors. It first establishes a Bingham model that describes properties of the hyper-concentrated flow. Next, it analyzes the relation between sediment parameters and flow parameters. Then, it obtains the relation between stability characteristics of the flow and such sediment parameters as sediment concentration, median diameter, size distribution and so on. Results of this research are also the theoretical basis to design pipelines carrying high concentrations of slurries.

3. Governing equations

The Yellow River is a wide-shallow river. According to the field data (Qian, 1989), the river is generally 500 m wide. The width can be as great as 1500 m when the river comes to its estuary. Even in its narrowest section, the width of the river is more than 50 m. However, the water depth of the river is quite small. The average depth is only 2.5 m, and in some shallow places, it is only 1.2–1.3 m. In such a wide-shallow river, the side-wall effects can be neglected.

Referring to the coordinate system shown in Fig. 1, X is in the stream-wise direction, Y is in the direction perpendicular to riverbed, Z is in the direction perpendicular to both X and Y, $J = \sin\theta$ is the slope of riverbed with θ as the slope angle of riverbed. The flow in the Yellow River can be treated as a two-dimensional flow on the X-Y plane. Since the constitutive relations carried out by many investigators are for the mixture of water and sediment (Fei, 1983; Fei and Yang, 1985; Qian, 1989; Fei, 1991; Wang, 2002), the governing equations of the water-sediment mixture are adopted in this research. Details are as follows,

Continuity equation:

$$\nabla\cdot\boldsymbol{u}_d = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial \boldsymbol{u}_d}{\partial T} + (\boldsymbol{u}_d\cdot\nabla)\boldsymbol{u}_d = (1-C)\boldsymbol{g} + C\rho_s\boldsymbol{g} - \frac{\nabla p_d}{\rho} + \frac{1}{\rho}\nabla\cdot\tau \tag{2}$$

where T is the time; C is the volumetric sediment concentration; $\boldsymbol{u}_d$ is the dimensional velocity vector; $\boldsymbol{g}$ is the gravitational acceleration vector; ρ is the density of water; ρ_s is the density of sediment; p_d is the pressure; $\boldsymbol{\tau}$ is the dimensional deviatoric stress tensor. The velocity vector $\boldsymbol{u}_d$ is in the form of $\boldsymbol{u}_d = u_d\boldsymbol{e_x} + v_d\boldsymbol{e_y}$, where u_d, v_d are the velocity components and $\boldsymbol{e_x}$, $\boldsymbol{e_y}$ are the unit vectors in X and Y directions, respectively. The gravitational acceleration is in the form of $\boldsymbol{g} = g\times\sin\theta\boldsymbol{e_x} - g\times\cos\theta\boldsymbol{e_y}$, where g is the gravitational acceleration. The volumetric sediment concentration C is different from the mass sediment concentration mentioned in the Introduction. Since the definition of the mass sediment concentration is the mass of the sediment contained in unit volume of turbid water, while that of the volumetric