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RESEARCH ARTICLE

Simple nonlinear model for the relationship between maize yield and cumulative water amount

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Abstract

Both the additive and multiplicative models of crop yield and water supply are polynomial equations, and the number of parameters increases linearly when the growing period is specified. However, interactions among multiple parameters occasionally lead to unreasonable estimations of certain parameters, which were water sensitivity coefficients but with negative value. Additionally, evapotranspiration must be measured as a model input. To facilitate the application of these models and overcome the aforementioned shortcomings, a simple model with only three parameters was derived in this paper based on certain general quantitative relations of crop yield (Y) and water supply (W). The new model, $Y/Y_{\mathrm{m}}–W^k/(W^k+w_{\mathrm{h}}^k)$, fits an S or a saturated curve of crop yield with the cumulative amount of water. Three parameters are related to biological factors: the yield potential (Y_{m}), the water requirement to achieve half of the yield potential (half-yield water requirement, w_{h}), and the water sensitivity coefficient (k). The model was validated with data from 24 maize lines obtained in the present study and 17 maize hybrids published by other authors. The results showed that the model was well fit to the data, and the normal root of the mean square error (NRMSE) values were 2.8 to 17.8% (average 7.2%) for the 24 maize lines and 2.7 to 12.7% (average 7.4%) for the 17 maize varieties. According to the present model, the maize water-sensitive stages in descending order were pollen shedding and silking, tasselling, jointing, initial grain filling, germination, middle grain filling, late grain filling, and end of grain filling. This sequence was consistent with actual observations in the maize field. The present model may be easily used to analyse the water use efficiency and drought tolerance of maize at specific stages.

Keywords: yield, water, model, maize, water sensitivity, drought tolerance

1. Introduction

The relationship between crop yield and water quantity is the foundation for optimizing irrigation schemes and developing new methods for evaluating drought tolerance, and research in this field can be traced back 100 years (Briggs 1914). Because a primary model for the whole growing period was established (Wit 1958), many models have been developed and continuously improved, and they include simple and

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complex models as well as empirical and theoretical models (Stewart *et al*. 1973, 1976; Blank 1975; Hanks 1975; Barrett 1980; Morgan *et al*. 1980; Singh *et al*. 1987; Rao *et al*. 1988; Li *et al*. 1997; Mehdi *et al*. 2014; Paredes *et al*. 2014; Shabani *et al*. 2015).

A simple model is more acceptable in application because of its convenience (Peng *et al*. 2000; Shen *et al*. 2001b). Additive and multiplicative models are relatively simple compared with complicated theoretical models (Xia *et al*. 2003). Typical additive models were developed by Blank (1975), Stewart *et al*. (1973, 1976) and Singh (1987), and multiplicative models were developed by Jensen (1973), Minhas *et al*. (1974), Hanks (1975), and Rao *et al*. (1988).

Additive models often have been criticized because they do not consider the lag effect of drought stress in crop growth stages, which may cause the water sensitivity coefficient to be negative (Taylor *et al*. 1983; Shen *et al*. 2001a, b). Therefore, multiplicative models, such as the Jensen model, have been a focus of greater attention (Rajput and Singh 1986; Wang *et al*. 2001; Cong *et al*. 2002; Wei *et al*. 2002; Zhang 2009). However, increasing evidence has indicated that these multiplicative models can also result in negative water sensitivity coefficient values (Li 1999; Jiao *et al*. 2004; Wei 2004), which may explain why the multiplicative model has been improved by certain researchers (Guo 1994; Chen *et al*. 1998; Cong *et al*. 2002; Jiao *et al*. 2004, 2005; Wei 2004).

In the mathematical structure of the models, both additive and multiplicative models include several parameters that are located at an equivalent position and have an equivalent meaning; thus, these parameters have the same statistical distribution (Wei 2004). A large change in one parameter may easily be obscured by slight variations of several other parameters if test errors are introduced (Jiao *et al*. 2004). When the growth stage is specified, the number of parameters in both models will increase, which also increases the risk of parameter offset. This offset may explain why a multi-parameter model can fit the experimental data well while containing unreasonable parameters. Compared with a linear model, a nonlinear model has few parameters that are not at equivalent positions, which allows this type of model to effectively avoid negative parameters.

The objective of this paper is to establish a simple nonlinear model for the relationship between crop yield and water supply and validate the model performance using data from 24 maize lines obtained in this study and 17 maize hybrids published by others.

2. Materials and methods

2.1. Model derivation and description

The derivation of the model was based on three basic assumptions. First, the crop yield (Y) does not increase indefinitely with water increases. With increases in accumulated irrigation (W), the relative increment of crop yield will decrease gradually. Therefore, the change rate of crop yield with the amount of water is proportional to $1/W$. Second, the increasing rate of Y with the amount of water depends on the developmental status of the plant, and it is lower at the early growing stage but higher at the late growing stage because the plant is larger and needs much more water. Thus, the change rate of the crop yield with the amount of water is proportional to the yield itself or the growing quantity (biomass). Third, when the plant develops to maturity and the yield reaches the maximum (Y_m), less water is required. Therefore, the change rate of the crop yield with the amount of water is proportional to $(1-Y/Y_\mathrm{m})$.

According to the above three assumptions, we established the following differential eq. (1):

$$\lambda=\frac{\partial Y}{\partial W}=k\times\frac{1}{W}\times Y\times\left(1-\frac{Y}{Y_\mathrm{m}}\right) \tag{1}$$

After integral derivation, we obtained an exponentiation saturation model for crop yield and water accumulation:

$$\frac{Y}{Y_\mathrm{m}}=\frac{W^k}{W^k+W_\mathrm{h}^k} \tag{2}$$

Where, Y is the yield (or dry matter) and W is the cumulative amount of water. k is a constant that is only related to the line/variety. For stage j, the cumulative amount of water is as follows:

$$W_j=\sum_{t=1}^{j}\omega_t$$

Where, ω_t is the actual individual amount of water at time t, which includes irrigation and rainfall.

The present model (eq. (2)) only has three parameters (Y_m, W_h, k), and they are located at different positions in the model structure and have different biological meanings.

Y_m is the maximum yield (an expression of productivity) if water is not a limiting factor.

W_h is the cumulative amount of water required to reach half of the maximum yield (half-yield water requirement). Within the same maturity period, smaller parameters correspond to stronger drought tolerance.

k is the water sensitivity coefficient. Lines/varieties with large k values have higher water use efficiency but weaker drought tolerance.

Generally, a line/variety with higher drought tolerance has small W_h and k values. Therefore, the combination form W_h^k or logic form $k\ln W_\mathrm{h}$ can be used for the evaluation of drought tolerance.

The characteristics of the present model must be described because certain characteristics will be used in subsequent applications.

1) According to eq. (2), the model fits an S curve if $k>1$ and a saturated curve if $k=1$.