Journal of Theoretical Biology 455 (2018) 7–15

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/jtbi

A solution for private assessment in indirect reciprocity using solitary observation

Isamu Okada[a,*], Tatsuya Sasaki[b,c], Yutaka Nakai[d]

[a] Soka University, Tokyo, Japan
[b] University of Vienna, Vienna, Austria
[c] F-Power Inc, Tokyo, Japan
[d] Shibaura Institute of Technology, Saitama, Japan

ARTICLE INFO

Article history:
Received 7 April 2018
Accepted 19 June 2018
Available online 3 July 2018

Keywords:
Evolution of cooperation
Indirect reciprocity
Private information
Evolutionary game theory

ABSTRACT

Although indirect reciprocity is a fundamental mechanism in the evolution of human cooperation, most studies assume public assessment in which individuals are not permitted to obtain private assessments of others. Existing studies on private assessment have used individual-based simulations because of the analytical difficulty involved. Here, we develop an analytical method using solitary observation to solve private assessment in indirect reciprocity problem without any approximation. In this study, we formulate a model of solitary observation and calculate the replicator dynamics systems of five leading norms of indirect reciprocity. Indirect reciprocity in private assessment provides a different result to that in public assessment. According to the existence proofs of cooperative evolutionarily stable (CES) points in the system, strict norms (stern judging and shunning) have no CES point in private assessment, while they do in public assessment. Image scoring does not change the system regardless of the assessment types because it does not use second-order information. In tolerant norms (simple standing and staying), the CES points move to co-existence of norms and unconditional cooperators. Despite the fact that there is no central coercive assessment system in private assessment, the average cooperation rate at the CES points in private assessment is greater than that in public assessment. This is because private assessment gives unconditional cooperators a role. Our results also show the superiority of the staying norm. Compared with simple standing, staying has three advantages in private assessment: a higher cooperation rate, easiness of invasion into defectors, and robustness to maintain cooperative evolutionarily stable situations. Our results are applicable to general social dilemmas in relation to private information. Under some dilemmas, norms or assessment rules should be carefully chosen to enable cooperation to evolve.

1. Introduction

Indirect reciprocity (Ghang and Nowak, 2015; Leimar and Hammerstein, 2001; Milinski et al., 2002; Nowak and Sigmund, 1998a; 1998b; 2005; Ohtsuki et al., 2009; Rockenbach and Milinski, 2006; Sasaki et al., 2016; Suzuki and Kimura, 2013) is a fundamental mechanism in the evolution of cooperation even in modern society which has highly mobile and large-scale relationships among individuals. Assessments of unrelated individuals are an essential mechanism for the avoidance of free riders, and thus it is considered that indirect reciprocity requires a shared reputation system that distributes images of all individuals in a society (Grimalda et al., 2016; McNamara and Doodson, 2015; Sommerfeld et al., 2007; Swakman et al., 2016). Therefore, the majority of theoretical studies on indirect reciprocity have assumed public assessment (Brandt and Sigmund, 2004; 2006; Chalub et al., 2006; Fishman, 2003; Leimar and Hammerstein, 2001; Masuda and Ohtsuki, 2007; Ohtsuki and Iwasa, 2004; 2006; 2007; Panchanathan, 2011; Panchanathan and Boyd, 2003; 2004; Sigmund, 2010; Watanabe et al., 2014). However, this is an over-simplification because an image of an individual is not necessarily the same for all individuals. Several studies have focused on private assessment (Martinez-Vaquero and Cuesta, 2013; Ohtsuki et al., 2015; Olejarz et al., 2015; Sigmund, 2012; Uchida, 2010; Uchida and Sasaki, 2013; Uchida and Sigmund, 2010), in which they relax such an unnatural assumption. Unfortunately, solving the problem of private assessment in indirect reciprocity is difficult because the number of equations goes

* Corresponding author.
E-mail address: okada@soka.ac.jp (I. Okada).

https://doi.org/10.1016/j.jtbi.2018.06.018
0022-5193/

to infinity. Thus, to date, studies on private assessment have used either numerical analysis or individual-based simulations (Okada et al., 2017; Yamamoto et al., 2017).

Previous studies on private assessment in indirect reciprocity using individual-based simulations have produced results that differ significantly from those related to public assessment. Okada et al. (2017) reveal three features: narrow and unstable cooperation, stable coexistence of discriminators and unconditional cooperators, and Pareto improvement. Although their pioneering work should be acknowledged, rigorous analysis using a theoretical solution is needed for general situations.

Here, we develop an analytical method using solitary observation to solve the problem of private assessment in indirect reciprocity. Any model of private assessment without restricted conditions cannot be solved analytically because this would require the solving of an infinite number of equations. This is because the definition of the conjunctive probability of v players whose images of a specified player are the same needs the conjunctive probability of $v+1$ players when the number of game observers is infinite. The extreme case of this restriction is a solitary observation in which the number of observers per game is set at one. Although this might be an extreme assumption, a theoretical comparison between public and private assessment is important for rigorous discussion.

In this study, we consider the five leading norms in indirect reciprocity studies: image scoring (Nowak and Sigmund, 1998b; Wedekind and Milinski, 2000), simple standing (Milinski et al., 2001; Sugden, 1986), stern judging (Kandori, 1992; Pacheco et al., 2006; Santos et al., 2016), shunning (Takahashi and Mashima, 2006), and staying (Nakai and Muto, 2005; 2008; Okada et al., 2017; Sasaki et al., 2017). We formulate a model of solitary observation and calculate the replicator dynamics systems of the five leading norms of indirect reciprocity. The analysis of a solitary observation shows that cooperative evolutionary stable strategies depend on the cost benefit ratio of a social dilemma game in private assessment, while this is not the case in public assessment. Our results confirm the insights of Okada et al. (2017).

2. Results

2.1. Giving games and solitary observation

We consider players who play giving (donation) games. A donor playing a game decides whether to contribute (C) or do nothing (N). If the donor chooses C, she or he must pay a fixed amount to a recipient. The recipient receives a benefit if and only if the donor contributes. We set the cost benefit ratio of a game to $r>1$. If the donor chooses N, nothing happens. Self-interested myopic players choose N because contributors do not benefit from their own contribution, and thus our model reveals a social dilemma. Players in the model consist of three norm-adopters: unconditional cooperators (X) who always contribute, unconditional defectors (Y) who never contribute, and discriminators denoted as Z. The discriminators have their private binary images (good or bad, denoted as G and B, respectively) of all the other players and only contribute to those whose images are good, thus they are called conditional cooperators.

All of the players face two types of errors: those in implementation and those in assessment. The donor defects in contradiction to her or his intention to contribute with a probability, e_1, and the observer oppositely mistakes the assessment for the donor with a probability, e_2. Let $\bar{e_1}=1-e_1$, $\bar{e_2}=1-e_2$, and $\epsilon=\bar{e_1}\bar{e_2}+e_1e_2$. For simplicity, we assume that $0<e<1/2<\epsilon<1$ is strictly satisfied where e is replaced by e_2.

In solitary observation, a game can only be observed by one observer. In each game, a donor (D), a recipient (R), and an observer (V) are randomly chosen. A player may fill one, two, or all three of those roles while the observer must be a Z player. In the game, only the observer is given a chance to update the donor's image, while the other discriminators have no such opportunity. Let $I_{jk}\in\{G, B\}$ be player k's image as assessed by player j. For example, I_{DR} is a recipient's image assessed by a donor in a game.

Table 1
Norms considered in this study.

Assessment rules	Donor's action (C/N) and Recipient's image (G/B)			
	C/G	N/G	C/B	N/B
Image scoring	G	B	G	B
Simple standing	G	B	G	G
Stern judging	G	B	B	G
Shunning	G	B	B	B
Staying	G	B	K	K

Note: "K" means that the discriminator does not update the donor's image regardless of one's play.

2.2. Replicator dynamics systems in the games

An infinite number of well-mixed players play games repeatedly using a continuous-entry model (Brandt and Sigmund, 2005). Let x, y and z be the fractions (population ratios) of X, Y, and Z, respectively, where $x+y+z=1$ is always satisfied. To explore the evolutionary dynamics of the private images, we consider an analysis of the marginal value of a good reputation (Ohtsuki et al., 2015), where no two players can ever meet more than once due to the assumption of an infinitely large population, and thus the chance of direct reciprocity is excluded from the model. In this framework, the time scale for natural selection is much slower than that of social interactions and image updates. Therefore, we can always assume that the frequency of good players is at its equilibrium value, that is, the expected probability that a players image is good has converged to a steady state. Then, we can define the parameters regarding the frequencies of good players in the steady state. Let g be the fraction of good players assessed by Z. This fraction is decomposed into g_x, g_y and g_z, where g_s is the fraction of good players with strategy s in the set $S=\{X,Y,Z\}$. $g=xg_x+yg_y+zg_z$ is always satisfied.

We consider a replicator equation system (Hofbauer and Sigmund, 1998). Let P_s be the expected payoff per the cost of the game of an s strategist where $s\in S$. The replicator dynamics are described as $\dot{x}=x(P_X-\bar{P})$, $\dot{y}=y(P_Y-\bar{P})$, and $\dot{z}=z(P_Z-\bar{P})$, where $\bar{P}=xP_X+yP_Y+zP_Z$ is the average payoff over the population. The expected payoffs of the three strategists are:

$$P_X = r(x+zg_x)-1$$
$$P_Y = r(x+zg_y)$$
$$P_Z = r(x+zg_z)-g.$$

where we omit the factor $\bar{e_1}$.

2.3. The five leading norms

In this study, we consider the five norms shown in Table 1. These norms have a common feature in terms of their assessment rules: contribution to a good recipient is assessed as good and defection to a good recipient is assessed as bad. The "image-scoring" norm is that cooperation is assessed as good, while defection is assessed as bad regardless of the recipient's image. The most tolerant norm, which is called "simple standing," is that both cooperation and defection to a bad recipient are assessed as good, while the stricter norm, which is called "stern judging," is that contribution to a bad recipient is assessed as bad while defection is assessed as good. The strictest norm is called "shunning," where any action to