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Asymmetric evolutionary game dynamics based on individuals' own volition

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ABSTRACT

For the pairwise interactions, the evolution of individual behavior should involve two major factors: one is what you will do in an interaction with a given opponent, and another is what type of opponents you prefer to interact with. In this study, we developed a two-phenotype iterated bimatrix replicator dynamics model based on individuals' own volition, where, different from the classic iterated game model, we assume that (*i*) for all interaction pairs, the maximum expected interaction time is same and it is limited even if two individuals in an interaction pair would like to keep their interaction; and (*ii*) all individuals are able to unilaterally break off the interactions with their opponents according to their own volition. Therefore, we define that, at any time t, an interaction pair will be disbanded with a given probability and the new interaction pairs will be randomly formed. The main results show that: (*i*) the existence of locally asymmetrically stable interior equilibrium is possible; and (*ii*) the evolutionary stability of the system is similar to the classic asymmetric evolutionary game. These results may provide a new insight for revealing the evolutionary significance of asymmetric game dynamics.

1. Introduction

It is well known that Dawkins' battle of the sexes is one of the most famous asymmetric games in evolutionary game theory (Dawkins, 1976). In this model, he considered a conflict between males and females concerning their respective shares in parental investment. At a fundamental level, this game is rigged against females by the fact that they produce relatively few, large gametes, whereas males produce many small gametes. Females are thereby much more committed and can less afford to lose a child. Thus males are in many cases in a better position to desert their female partners and children. They can invest into increasing their offspring with the help of new mates. As an imaginary game, Dawkins assumed that there are two phenotypes (or called the strategies) in female population, called "coy" and "fast", respectively, and two phenotypes in male population, called "faithful" and "philander", respectively. The coy females insist on a long courtship, whereas the fast female do not; all females care for the offspring they produce. The faithful males are willing, if necessary, to engage in a long courtship, and also care for the offspring. The philandering males are not prepared to engage in a long courtship, and do not care for their offspring. All of these descriptions and logical reasoning imply a cyclical character: if females are coy, it pays males to be faithful; if males are faithful, it pays females to be fast; if females are fast, it pays males to be philandering; and if males are philandering, it pays females to be coy (Dawkins, 1976; Hofbauer and Sigmund, 1998; Maynard Smith, 1982; Schuster and Sigmund, 1981; Schuster et al., 1981).

In order to model the battle of the sexes theoretically, assume that: (*i*) the successful raising of an offspring increase the fitness of both parents by G; (*ii*) the parental investment $-C$ will be entirely borne by the female if the male deserts, otherwise, it will be shared equally by both parents; (*iii*) a long engagement period represents a cost of $-E$ to both partners; and (*iv*) $0 < E < G < C < 2(G - E)$ (Hofbauer and Sigmund, 1998). Thus, if a faithful male mates with a coy female, the payoff is $G - C/2 - E$ for both; if faithful male mates with a fast female, the payoff is $G - C/2$ for both; if a philandering male encounters a coy female, the payoff is 0 for both; and if a philandering male mates with a fast female, the payoffs of the male and female are G and $G - C$, respectively (Hofbauer and Sigmund, 1998). The analysis of the asymmetric replicator dynamics (or the bimatrix replicator dy-

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namics) shows clearly that the battle of the sexes has an interior equilibrium (or a unique mixed Nash equilibrium), which is $\left(x^* = E/(C-G+E),\ \ y^* = C/2(G-E)\right)$ (where x denotes the proportion (or frequency) of the phenotype "philander" in the male population and y the proportion of the phenotype "coy" in the female population), but it is unstable and all orbits are periodic orbits surrounding it (Hofbauer and Sigmund, 1998; Schuster and Sigmund, 1981; Schuster et al., 1981). Although this result provides a possible theoretical explanation for the evolution of the battle of the sexes, a challenging question is whether the cyclic character of the battle of the sexes is really favored by natural selection? As stated by Maynard Smith, "I am unable to offer illustrative examples, or evidence that such cycles occur" (Maynard Smith, 1982).

In fact, we also notice that the classic theory of replicator dynamics has shown that for a two-phenotype bimatrix game dynamics, if a unique interior equilibrium exists, then it must be unstable, or it is either saddle point or singular point (Hofbauer and Sigmund, 1998; Schuster and Sigmund, 1981; Schuster et al., 1981). In a standard bimatrix game, it is assumed that: (*i*) players in different positions (for example, males and females in the battle of the sexes) have different strategy sets and payoff matrices, where for convenience, two positions are denoted by position I and position II, respectively, and two strategy sets are denoted by $\mathbf{R} = \left(R_1, R_2, \cdots, R_n\right)$ for position I and $\mathbf{S} = \left(S_1, S_2, \cdots, S_m\right)$ for position II; and (*ii*) at any time t, for each interaction pair $(R_i,\ S_j)$, the probability that a R_i-player interacts with a S_j-player exactly equals to the frequency of S_j in the II-population, and similarly, the probability that a S_j-player interacts with a R_i-player is the frequency of R_i in the I-population for all $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$. The second assumption strongly implies that the pairwise interactions between I-players and II-players should be uniform. However, this assumption may be not always true in nature. In 2006, Taylor and Nowak developed the concept of non-uniform interaction rates in evolutionary symmetric game dynamics, i.e. the probability of interaction between two individuals is not independent of their strategies, and they showed that the non-uniform interaction rates allow the coexistence of cooperation and defection in Prisoner's Dilemma (PD) game (Taylor and Nowak., 2006). In fact, the concept of non-uniform rates also implies that under the framework of evolutionary game with pairwise interactions, the evolution of behavior should involve two major factors: one is what you will do in an interaction with a given opponent, and another is what type of opponents you prefer to interact with. Obviously, both of these two factors should depend on individuals' self-interests. Recently, an interesting experimental study shows clearly that, based on individual self-interest in PD game, if each player can unilaterally break off the pairwise interaction with his/her opponent according to his/her own volition, then all individuals (including both cooperators and defectors) prefer a cooperator as an opponent (Zhang et al., 2016; Zheng et al., 2017). So, this simple mechanism should have very important evolutionary significance in evolutionary game dynamics (Eshel and Cavalli-Sforza, 1982; Fletcher and Doebeli., 2006). However, we here have to point out that the mechanism based on individuals' own volition, proposed by Zhang et al. (2016) and Zheng et al. (2017), is totally different from the concept of "assortative matching" (Chaudhuri, 2011) since in their models they assumed that the population size is large enough (or infinite) and individuals are unable to detect the strategies of their opponents before they interact with their opponents.

In this study, we will extend the basic idea in Zhang et al. (2016) and Zheng et al. (2017) to the bimatrix game dynamics. For example, according to the definitions and assumptions of the battle of the sexes with payoff matrices $\begin{pmatrix} 0 & G \\ G-\frac{C}{2}-E & G-\frac{C}{2} \end{pmatrix}$ (for male) and $\begin{pmatrix} 0 & G-\frac{C}{2}-E \\ G-C & G-\frac{C}{2} \end{pmatrix}$ (for female) (Hofbauer and Sigmund, 1998), both faithful and philandering males should more prefer to mate with a fast female, and both coy and fast females should more prefer to mate a faithful male. However, our main goal is not merely limited to the conflict between the two sexes, but rather to consider more generally the effect of the individuals' own volition on the dynamics of two-phenotype asymmetric game.

2. Assumptions and model

Consider a two-phenotype asymmetric game (or a two-phenotype bimatrix game) between R-individuals and S-individuals, where two phenotypes in R-population are denoted by R_1 and R_2, respectively, and two phenotypes in S-population are denoted by S_1 and S_2, respectively. The two payoff matrices are given by $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, where b_{ij} denotes the payoff of a R_i-individual in unit time when it plays against a S_j-individual for $i, j = 1, 2,$ and a_{ji} is the payoff of a S_j-individual in unit time when it plays against a R_i-individual for $j, i = 1, 2$ (Hofbauer and Sigmund, 1998).

Furthermore, we make some basic assumptions, which are (*i*) both R- and S-populations are large enough; (*ii*) the interaction between R- and S-individuals is an iterated game (but it is different from the classic iterated game), where, similar to Zhang et al. (2016) and Zheng et al. (2017), we assume that for all interaction pairs, the maximum expected interaction time (or the maximum number of the repeated rounds) is a same fixed constant, all individuals are able to unilaterally break off the interactions with their opponents at any time t according to their own volition, and the new interaction pairs are randomly formed; (*iii*) similar also to Zhang et al. (2016) and Zheng et al. (2017), all individuals are assumed to be unable to identify the strategies of their opponents before they interact with the opponents. Based on these assumptions, we define that at any time t an interaction pair between R- and S-individuals will be disbanded with a given probability, where this probability depends on both R-individual's strategy and S-individual's strategy.

Let P_{ij} be the frequency of interaction pair $(R_i,\ S_j)$ for $i, j = 1, 2$. Then, the frequencies of R_1 and R_2 in the R-population, denoted by x and $1-x$, respectively, are given by

$$\begin{aligned} x &= P_{11} + P_{12}\ , \\ 1-x &= P_{21} + P_{22}\ , \end{aligned} \tag{1}$$

and, similarly, the frequency of S_1 and S_2 in the S-population, denoted by y and $1-y$, respectively, are given by

$$\begin{aligned} y &= P_{11} + P_{21}\ , \\ 1-y &= P_{12} + P_{22}\ . \end{aligned} \tag{2}$$

The interaction pair $(R_i,\ S_j)$ will be disbanded with probability ϕ_{ij} at any time t for $i, j = 1, 2$ and that the single R-individuals and single S-individuals will form new interaction pairs through the random meeting (Taylor and Nowak., 2006). Let n_{ij} denote the number of interaction pair $(R_i,\ S_j)$ for $i, j = 1, 2$ and notice that

$$p = \frac{\sum_j \phi_{1j} n_{1j}}{\sum_{i,j} \phi_{ij} n_{ij}} \tag{3}$$

is the proportion of R_1 in the single R-group, and

$$q = \frac{\sum_i \phi_{i1} n_{i1}}{\sum_{i,j} \phi_{ij} n_{ij}} \tag{4}$$

is the proportion of S_1 in the single S-group. Then, the change rates of n_{ij} $(i, j = 1, 2)$ because of the recombination of interaction pairs