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A reduced complexity model of a gravel-sand river bifurcation: Equilibrium states and their stability

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ABSTRACT

We derive an idealized model of a gravel-sand river bifurcation and analyze its stability properties. The model requires nodal point relations that describe the ratio of the supply of gravel and sand to the two downstream branches. The model predicts changes in bed elevation and bed surface gravel content in the two bifurcates under conditions of a constant water discharge, sediment supply, base level, and channel width and under the assumption of a branch-averaged approach of the bifurcates. The stability analysis reveals more complex behavior than for unisize sediment: three to five equilibrium solutions exist rather than three. In addition, we find that under specific parameter settings the initial conditions in the bifurcates determine to which of the equilibrium states the system evolves. Our approach has limited predictive value for real bifurcations due to neglecting several effects (e.g., transverse bed slope, alternate bars, upstream flow asymmetry, and bend sorting), yet it provides a first step in addressing mixed-size sediment mechanisms in modeling the dynamics of river bifurcations.

1. Introduction

River bifurcations or diffluences are found in alluvial fans, braided rivers, anabranching rivers, deltas, cut-off channels, diversions (for flood control or water intakes), and in constructed side channels that are part of river restoration schemes. Once a bifurcation is initiated, a downstream channel (or bifurcate or distributary) continues to deepen as long as the sediment transport capacity exceeds the sediment supply to the channel.

Sediment transport in a channel consists of bed-material load (i.e., bed load and suspended bed-material load) and wash load (Church, 2006; Paola, 2001). As wash load is typically assumed to be distributed uniformly over the water column, it is assumed to be partitioned over the bifurcates according to the ratio of the water discharge. Bed-material load, however, partitions over the bifurcates in a less straightforward manner. The partitioning of sediment in streams dominated by suspended bed-material load depends on the initial flow depth and channel slope in the bifurcates (Slingerland and Smith, 1998), the grain size of the bed sediment (Slingerland and Smith, 1998), and curvature-induced effects in the upstream channel (Hackney et al., 2017). The partitioning of sediment in bed load dominated streams depends on:

- the conditions in the bifurcates: base level, channel width, friction, bifurcation angle (Bulle, 1926; Van der Mark and Mosselman, 2013; Tarekul Islam et al., 2006), and the zones of flow recirculation close to the bifurcation (Bulle, 1926; De Heer and Mosselman, 2004; Marra et al., 2014; Thomas et al., 2011), vegetation (Burge, 2006), and cohesive sediment and bank erosion (Miori et al., 2006; Zolezzi et al., 2006);
- the conditions in the area just upstream of the bifurcation: the transverse distribution of water and sediment over the upstream channel, which is affected by secondary flow (Van der Mark and Mosselman, 2013), a transverse bed slope induced by an inlet step (Bolla Pittaluga et al., 2003), alternate bars (Bertoldi and Tubino, 2007; Bertoldi et al., 2009; Redolfi et al., 2016), and sediment mobility (Frings and Kleinhans, 2008);
- conditions extending further upstream: flow asymmetry induced by a bend, which tends to provide one bifurcate with a larger fraction content of the flow and the other one with a larger fraction content of the sediment load (Federici and Paola, 2003; Hardy et al., 2011; Kleinhans et al., 2008; Van Dijk et al., 2014) and transverse sediment sorting due to bend flow (Frings and Kleinhans, 2008; Sloff et al., 2003; Sloff and Mosselman, 2012).

The partitioning of the sediment load over the bifurcates determines whether the bifurcation develops towards a stable state with two open downstream branches or a state in which the water discharge in one of the branches continues to increase at the expense of the other branch. The latter case may lead to the silting up of one of the downstream

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channels. Under such conditions a one channel configuration is a stable equilibrium solution of the bifurcation system (Wang et al., 1995), yet in literature this situation is often termed an 'unstable bifurcation' (Burge, 2006; Federici and Paola, 2003), as the two channel system ceases to exist.

Early one-dimensional reduced complexity models describing the development towards the equilibrium states of two bifurcates have been developed for bed load transport in sand-bed rivers (Wang et al., 1995), bed load transport in gravel-bed rivers (Bolla Pittaluga et al., 2003), and suspended bed-material load (Slingerland and Smith, 1998). Such reduced complexity models allow for the computation of the partitioning of the water discharge as the water surface elevation at the bifurcation point must be equal between the three reaches. The sediment partitioning, however, depends on the geometry of the bifurcation and the three-dimensional flow structure, which obviously cannot be reproduced by a one-dimensional model. A one-dimensional model therefore requires a nodal point relation that describes the partitioning of the sediment load over the bifurcates.

Wang et al. (1995) were the first to introduce a nodal point relation describing the partitioning of the sediment supplied from upstream over the bifurcates. They then apply a simpler form of their nodal point relation ($\bar{s}_1/\bar{s}_2 = (q_1/q_2)^k$, where $\bar{s}_{1,2}$ is the rate of sediment supply per unit width to branches 1 and 2 and $q_{1,2}$ is the water supply per unit width to branches 1 and 2) to analyse the stability of the solutions to the equilibrium morphodynamic state of the bifurcates. Studying avulsion development (Slingerland and Smith, 1998) introduce a nodal point relation that originates from integration of the vertical concentration profile of the suspended sediment. Alternative nodal point relations have been developed by Bolla Pittaluga et al. (2003), who account for the effects of a transverse bed slope that induces lateral sediment transport to the deeper bifurcate, and Kleinhans et al. (2008), who account for the effects of an upstream bend, both of which will be addressed in further detail below.

Pioneering work on bifurcation dynamics using a nodal point relation was conducted by Wang et al. (1995): they assume a constant water discharge and sediment supply rate in the upstream channel, a constant and equal base level in the two bifurcating branches, and unisize sediment conditions. They apply the Engelund and Hansen (1967) sediment transport relation without a threshold for significant transport: $s \propto U^n$, where s denotes the sediment transport capacity per unit width, U the depth-averaged flow velocity, and n is the exponent in the power law load relation ($n = 5$). They find that for $k < n/3$ the equilibrium solution where one of the bifurcates closes is stable, whereas for $k > n/3$ the equilibrium solution with two open branches is stable. Despite these early results a model for k is still lacking. Also Slingerland and Smith (1998) reveal that a bifurcation or avulsion develops towards a stable state with two open downstream branches or a state in which one channel becomes the dominant channel at the expense of the other branch.

In such strongly idealized one-dimensional analyses, two-dimensional and three-dimensional effects near the bifurcation point are not readily accounted for. One of these effects is the Bulle effect (Bulle, 1926; Dutta et al., 2017; Van der Mark and Mosselman, 2013), which indicates a situation where the sediment supply to a diversion channel (i.e., a channel that branches off the main channel under a certain angle) is significantly larger than the diversion channel's fraction content of the water discharge. This effect is associated with secondary flow (e.g., Thomas et al., 2011). Another effect is the difference in bed elevation that is associated with a difference in flow depth between the two bifurcates (e.g., Bolla Pittaluga et al., 2003; Kleinhans et al., 2013). This bed elevation difference (also denoted using the term inlet step) tends to increase the sediment supply to the deeper bifurcate (e.g., Slingerland and Smith, 1998), which acts as a stabilizing mechanism.

Although mixed-size sediment systems may reveal behavior that is essentially different from unisize sediment systems (Blom et al., 2017a; 2017b; 2016; Mosselman and Sloff, 2008; Sinha and Parker, 1996), so far the influence of noncohesive mixed-size sediment on bifurcation dynamics has not been studied explicitly. Wash load, suspended bed-material load, and bed load (Church, 2006; Paola, 2001) are expected to respond differently to the above-mentioned mechanisms (Hackney et al., 2017). Mixed-size sediment effects are the following:

1. As the vertical profile of sediment concentration is less uniform over depth for coarse sediment (i.e., coarse sediment tends to concentrate more strongly near the bed), coarse sediment tends to be affected more by an inlet step than fine sediment (Slingerland and Smith, 1998).
2. The effect of the transverse bed slope on lateral transport upstream of the bifurcation depends on grain size, where coarse sediment is affected by the transverse bed slope more strongly than fines (Parker and Andrews, 1985);
3. The presence of a bend upstream of the bifurcation typically leads to bend sorting and a coarser sediment supply to the distributary in the outer bend than to the one in the inner bend (Frings and Kleinhans, 2008; Sloff et al., 2003; Sloff and Mosselman, 2012);
4. Alternate bar formation and geometry appear to be affected by the grain size distribution of the sediment mixture (Bertoldi and Tubino, 2005; Lanzoni, 2000).

Our objective is to assess the elementary consequences of the introduction of mixed-size sediment mechanisms in the modelling of the dynamics of a river bifurcation. To this end we follow Wang et al. (1995)'s approach and its simple nodal point relation with associated limitations and simplifications: we neglect the effects of vegetation, cohesive sediment, bank erosion, alternate bars or a bend in the upstream channel, as well as the Bulle effect and the transverse slope effect. We extend their model to conditions with bed-material load of a two-fraction sediment mixture consisting of gravel and sand. This implies the need for two nodal point relations describing the ratio of, respectively, the gravel and sand supply to the two bifurcates. We study the stability of the equilibrium states of the bifurcates in an engineered river characterized by a fixed channel width.

The proposed analysis and model are applicable to both cases shown in Fig. 1: a bifurcation system with two bifurcates that are characterized by the same base level and a side channel system. We set up a model describing the equilibrium solutions of the mixed sediment bifurcation system (Section 2), we determine its equilibrium solutions (Section 3), we derive a system of ordinary differential equations for the flow depth and bed surface texture in the bifurcates (Section 4), and perform a stability analysis of the equilibrium solutions (Section 5). The analysis also provides insight on the time scale of the evolution towards the stable equilibrium solutions (Section 6).

2. Model of the equilibrium state

In this section we strongly simplify the situation of a gravel-sand river bifurcation, describe the problem from a mathematical point of view, and list the governing equations. To this end we consider an engineered river with a fixed channel width that may vary between the branches, a temporally constant water discharge in the upstream branch (i.e., branch 0 in Fig. 1) and a temporally constant gravel supply rate and constant sand supply rate to the upstream branch.

Under equilibrium conditions ($\partial/\partial t = 0$) without subsidence, uplift, and particle abrasion, the equation describing conservation of sediment mass (i.e. the Exner equation) reduces to the stationary Exner equation, $\partial S_i/\partial x = 0$, where S_i denotes the sediment transport capacity in branch i, the subscript i indicates branch i, and x is the streamwise coordinate. In other words, by definition the sediment transport capacity S_i equals the sediment supply to branch i, $\bar{S}_i$, where the bar indicates the sediment supply.

For simplicity we apply the Engelund and Hansen power law load relation (Engelund and Hansen, 1967):

$$S_i = B_i m_i U_i^n \tag{1}$$