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# Porous gravity currents: Axisymmetric propagation in horizontally graded medium and a review of similarity solutions

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ABSTRACT

We present an investigation on the combined effect of fluid rheology and permeability variations on the propagation of porous gravity currents in axisymmetric geometry. The fluid is taken to be of power-law type with behaviour index $n$ and the permeability to depend from the distance from the source as a power-law function of exponent $\beta$. The model represents the injection of a current of non-Newtonian fluid along a vertical bore hole in porous media with space-dependent properties. The injection is either instantaneous ($\alpha = 0$) or continuous ($\alpha > 0$). A self-similar solution describing the rate of propagation and the profile of the current is derived under the assumption of small aspect ratio between the current average thickness and length. The limitations on model parameters imposed by the model assumptions are discussed in depth, considering currents of increasing/decreasing velocity, thickness, and aspect ratio, and the sensitivity of the radius, thickness, and aspect ratio to model parameters. Several critical values of $\alpha$ and $\beta$ discriminating between opposite tendencies are thus determined. Experimental validation is performed using shear-thinning suspensions and Newtonian mixtures in different regimes. A box filled with ballotini of different diameter is used to reproduce the current, with observations from the side and bottom. Most experimental results for the radius and profile of the current agree well with the self-similar solution except at the beginning of the process, due to the limitations of the 2-D assumption and to boundary effects near the injection zone. The results for this specific case corroborate a general model for currents with constant or time-varying volume of power-law fluids propagating in porous domains of plane or radial geometry, with uniform or varying permeability, and the possible effect of channelization. All results obtained in the present and previous papers for the key parameters governing the dynamics of power-law gravity currents are summarized and compared to infer the combinations of parameters leading to the fastest/lowest rate of propagation, and of variation of thickness and aspect ratio.



## 1. Introduction

The propagation of gravity-driven flows in porous media is but a chapter of the fascinating 'book' on gravity currents (hereinafter GCs), which has received considerable attention (Ungarish, 2009), with new 'chapters' being continuously added. Yet also porous GCs by themselves, originating from such diverse applications (carbon dioxide sequestration, mining engineering, environmental pollution and remediation, seawater intrusion, to name but a few) constitute such a wide topic that a comprehensive summary is arduous. In the authors' view, the recent advancements on gravity-driven porous flow belong to two broad categories.

The first group of contributions has as a common feature the modelling of the spatial variations of properties and/or of boundary conditions in natural (geologic) media, and the description of their topographical features. Examples of such contributions are Huppert et al. (2013), Sahu and Flynn (2017), and Ngo et al. (2016), where heterogeneity is modelled via discrete layers or intrusions of finite extent; Islam et al. (2016), who introduce explicitly small-scale heterogeneity; Yu et al. (2017), who account simultaneously for drainage from a permeable substrate and an edge; and Huber et al. (2016), who aim at reproducing the effect of diverse $CO_2$ injection strategies.

The second broad group of GC-themed contributions presents an improved description of fundamental mechanisms via a more sophisticated modelling. Some relevant examples are the effects of a change in flux (Ball et al., 2017) or of stratification in an intruding current (Pegler et al., 2016); the investigation of the $CO_2$ sequestration mechanisms into deep saline aquifers, involving two-phase

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flow (Guo et al., 2016) or with possible background hydrological flow (Unwin et al., 2016); the interactions between gravity currents and convective dissolution (Elenius et al., 2015), or geomechanics (Bjornara et al., 2016); the adoption of realistic rheological models in the study of non-Newtonian GCs (Di Federico et al., 2017).

Some recent contributions belong to both categories, and are associated, for example, with the modelling of $CO_2$ sequestration (Ngo et al., 2016) or the simultaneous presence of non-Newtonian flow and spatial heterogeneity or specific topographical features. The latter topic has been investigated in depth in several papers, considering deterministic heterogeneity and radial (Di Federico et al., 2014) or plane geometry (Ciriello et al., 2016), and channelized flow (Di Federico et al., 2014). The motivation for these studies lies in a multiplicity of applications involving complex fluids flowing in geologic media characterized by spatial heterogeneity at various scales: oil and displacing suspensions in reservoir flow, remediation carriers and liquid contaminants in the subsurface environments, drilling and grouting fluids; earlier works (Ciriello et al., 2016; Di Federico et al., 2014) list specific references to these applications.

Studies of flows of non-Newtonian GCs rely on a body of knowledge accumulated for Newtonian currents: the reference works of Huppert and Woods (1995) for plane geometry, and by Lyle et al. (2005) for axisymmetric geometry, were extended to power-law fluid flow by Di Federico et al. (2012a,b), which, in turn, set the stage for the more complex setups cited earlier. Variations of properties along vertical and horizontal direction were considered in the context of Newtonian GCs by Zheng et al. (2014, 2013). While vertical variations mimic the effect of stratification, horizontal variations may represent e.g. the effect induced by the drilling of a well, and thus are of interest especially in the context of axisymmetric propagation. A review of existing studies on non-Newtonian porous GCs reveals the lack of a study coupling power-law rheology and permeability gradients along the flow direction in axisymmetric flow. Such a study is presented here in Sections 2–5 considering the usual hypothesis of a GC of time-variable inflow.

The exposition is organized as follows. The mathematical problem is formulated in Section 2 for a radial injection, and solved in Section 3 in self-similar form generalizing the results of Di Federico et al. (2012b). Section 4 discusses the dependency of key time exponents governing the propagation of the current on problem parameters, along with the limitations imposed by modelling assumptions. Experimental results are presented in Section 5; first, the experimental set-up is described, with special attention on the difficulties implied by simulating heterogeneity; then results from a series of tests are compared with the theory in constant- and variable-flux regime.

The theory and experiments presented complete a first picture on porous gravity currents of power-law fluid flowing in different geometries (plane and axisymmetric) in domains exhibiting permeability variations in different directions (vertical and horizontal), taking into account the influence of the channel cross section for plane flow. A general overview and comparison of these self-similar solutions seems timely, and is presented in Section 6. Concluding remarks are formulated in Section 7 together with perspectives for future work.

## 2. Problem formulation

Consider a non-Newtonian power-law fluid of density $\rho$, consistency index $m$, and flow behaviour index $n$, that spreads axisymmetrically over a horizontal bed into a porous medium of height $h_0$, initially saturated with a lighter fluid of density $\rho - \Delta\rho$ (see Fig. 1). Under the sharp interface and thin current approximations, and in the absence of capillary effects (see the recent paper by Chiapponi, 2017 for an indication of the fluid retention in a glass beads porous medium), the pressure within the current is hydrostatic, and given by $p(r,z,t) = p_1 + \Delta\rho\, gh(r,t) - \rho gz$, where $r$ and $z$ represent radial and vertical coordinates, $p_1 = p_0 + (\rho - \Delta\rho)gh_0$ is a constant, $p_0$ is the constant pressure at $z = h_0$, and $g$ is gravity. Under the additional assumption that the current thickness is small compared to that of the ambient fluid, the velocity of the latter and the vertical velocity in the intruding fluid can be neglected, allowing to describe the current behaviour by means of its horizontal velocity $u(r,t)$, thickness $h(r,t)$ and maximum extension $r_N(t)$ for given time $t$. The expression of the horizontal velocity can be deduced from the following general equation, valid for the motion of a power-law fluid in a porous medium (Cristopher and Middleman, 1965)

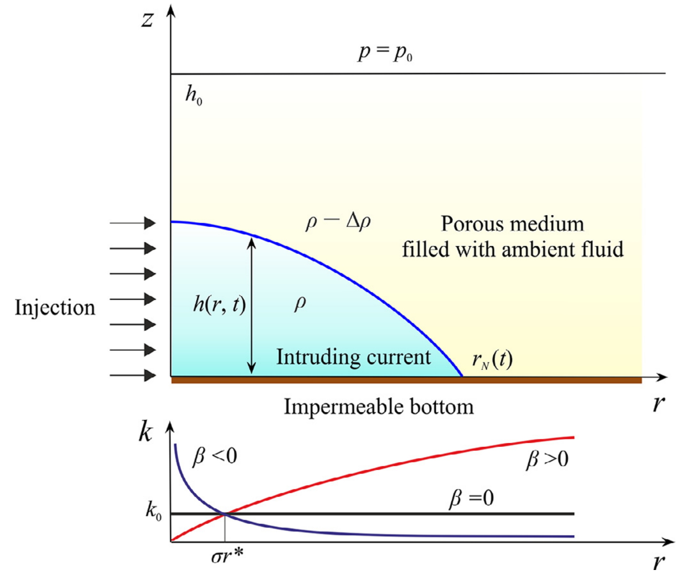


**Fig. 1.** Sketch of an axisymmetric gravity current intruding into a saturated porous medium of thickness $h_0$. The bottom panel illustrates radially increasing ($\beta > 0$), decreasing ($\beta < 0$), and homogeneous ($\beta = 0$) permeabilities.

$$\nabla p - \rho \boldsymbol{g} = -\frac{\mu_{eff}}{k}|\boldsymbol{u}|^{n-1}\boldsymbol{u}, \tag{1}$$

in which $p$ is the pressure, $\boldsymbol{u}$ is the Darcy velocity field, $\boldsymbol{g}$ is the gravity vector, $k$ the permeability, and $\mu_{eff}$ is the effective viscosity (dimensions $[\mathrm{ML}^{-n}\mathrm{T}^{n-2}]$). The mobility $\frac{\mu_{eff}}{k}$ is given by (Di Federico et al., 2012b)

$$\frac{k}{\mu_{eff}} = \frac{1}{2C_t}\frac{1}{m}\left(\frac{n\phi}{3n+1}\right)^n\left(\frac{50k}{3\phi}\right)^{(n+1)/2}, \tag{2}$$

where $\phi$ is the porosity and $C_t = C_t(n)$ the tortuosity factor. The modified Darcy's law (1) is based on a capillary bundle model first proposed by Bird et al. (1960) and later modified to include tortuosity, for which different formulations are available (e.g. Shenoy, 1995); in the following, the formulation by Pascal (1983), $C_t = (25/12)^{(n+1)/2}$, is adopted. Macroscopic laws having the same structure of Eq. (1) were obtained via direct simulation at the pore scale by e.g. Balhoff and Thompson (2006) and Vakilha and Manzari (2008). Experimental verification was provided, among others, by Cristopher and Middleman (1965) and Yilmaz et al. (2009). Additional references on the use of Eq. (1) are reported in Di Federico et al. (2012b). The model is unable to handle viscoelastic effects and thixotropy, and needs to be modified to include yield stress or Newtonian behaviour at low shear rates.

Local mass conservation implies that

$$\frac{1}{r}\frac{\partial}{\partial r}(ruh) = -\phi\frac{\partial h}{\partial t}, \tag{3}$$