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A residual-based shock capturing scheme for the continuous/discontinuous spectral element solution of the 2D shallow water equations

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ABSTRACT

The high-order numerical solution of the non-linear shallow water equations is susceptible to Gibbs oscillations in the proximity of strong gradients. In this paper, we tackle this issue by presenting a shock capturing model based on the numerical residual of the solution. Via numerical tests, we demonstrate that the model removes the spurious oscillations in the proximity of strong wave fronts while preserving their strength. Furthermore, for coarse grids, it prevents energy from building up at small wave-numbers. When applied to the continuity equation to stabilize the water surface, the addition of the shock capturing scheme does not affect mass conservation. We found that our model improves the continuous and discontinuous Galerkin solutions alike in the proximity of sharp fronts propagating on wet surfaces. In the presence of wet/dry interfaces, however, the model needs to be enhanced with the addition of an inundation scheme which, however, we do not address in this paper.

1. Introduction

The shallow water equations (SW) (de Saint-Venant, 1871) are a common $(d-1)$ approximation to the d-dimensional Navier–Stokes equations to model incompressible, free surface flows. Due to the ability of high-order Galerkin methods to keep dissipation and dispersion errors low (Ainsworth et al., 2006) and their flexibility with arbitrary geometries and *hp*-adaptivity, these methods are proving their mettle for solving the shallow water equations in the modeling of non-linear waves in different geophysical flows (Chun and Eskilsson, 2016; Dawson and Aizinger, 2005; Eskilsson, 2011; Gerhard et al., 2015; Giraldo, 2001; Giraldo et al., 2002; Giraldo and Restelli, 2010; Hendricks et al., 2016; Iskandarani et al., 1995; Kärnä et al., 2011; Kesserwani and Liang, 2012a; 2012b; Kubatko et al., 2006; Li et al., 2018; Ma, 1993; Marras et al., 2015; Nair et al., 2007; Taylor et al., 1997; Xing et al., 2010). One important property that high-order Galerkin methods offer and that makes them attractive over their low-order counterparts is given by their natural strong scaling properties on massively parallel computers (Abdi et al., 2016; Gandham et al., 2015; Müller et al., 2016). Nevertheless, the high-order solution of non-linear wave problems via high-order methods is susceptible to unphysical Gibbs oscillations that form in the proximity of strong gradients such as propagating bores. Filters like Vandeven's (1991)) and Boyd's (1996)) and different types of artificial viscosities are the most common tools to handle this problem for continuous and discontinuous Galerkin (CG/DG) methods. However, filtering may not be sufficient as the flow strengthens and the wave sharpness intensifies; for this reason, previous studies have stabilized the Galerkin solution to the shallow water equations in a variety of ways. For example, the Lilly–Smagorinsky eddy viscosity model (Lilly, 1962; Smagorinsky, 1963) was utilized in Phan Van et al. (2014) and Rakowsky et al. (2013) to preserve numerical stability without compromising the overall quality of the solution. To account for sub-grid scale effects, artificial viscosity was utilized in the DG model described in Gourgue et al. (2009) to improve their inviscid simulations. Recently, in Pasquetti et al. (2015), the high-order spectral element solution of the one-dimensional shallow water

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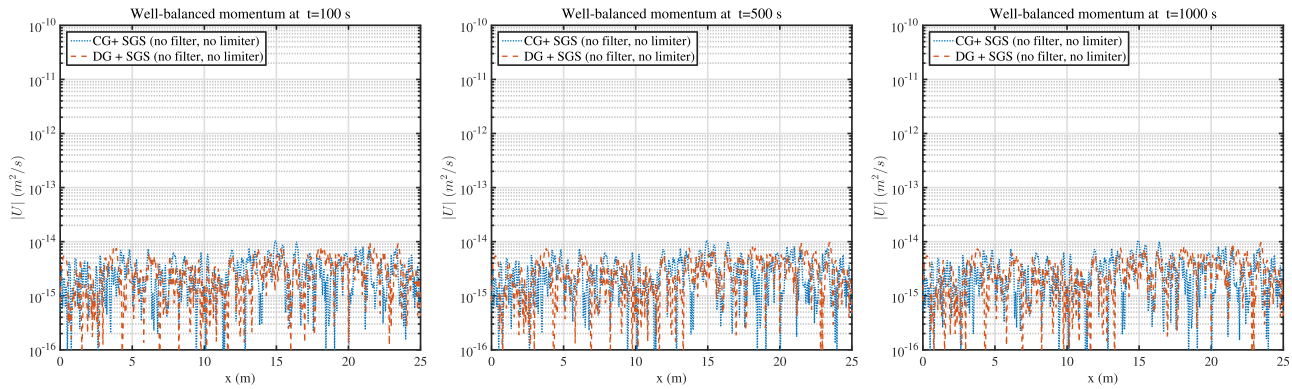

Fig. 1. Well-balanced CG and DG solutions of a lake at rest over a submerged hump. From left to right, the discharge is plotted at $t = [100, 500, 1000]$ s.

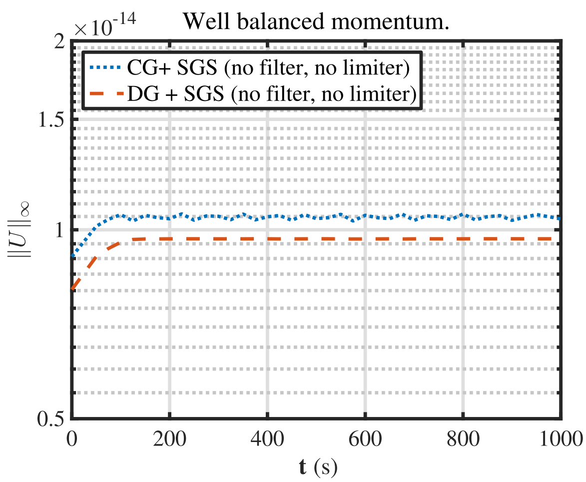

Fig. 2. Time evolution of the infinity norm of momentum for the well-balanced results plotted in Fig. 1.

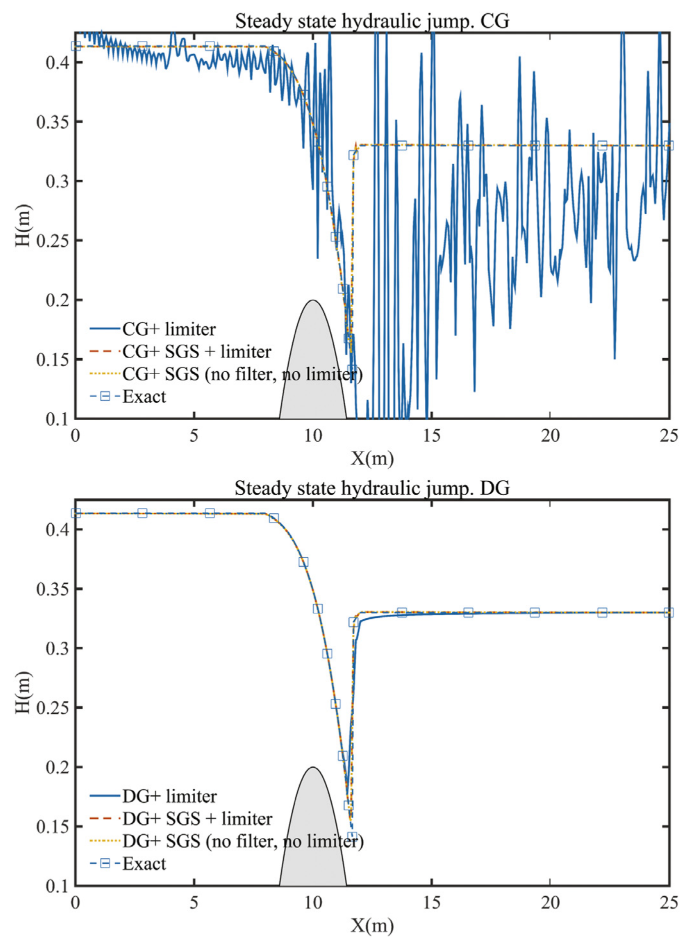

Fig. 3. Water surface computed with CG and DG for the steady state transcritical flow with a shock.

equations was stabilized via the entropy viscosity method. Artificial viscosity, limiters, and filters for the (modal) DG solution of SW were recently compared in Michoski et al. (2016), concluding that a dynamically adaptive viscosity may be the most effective means of regularization at higher orders.

Building on some of the insights from the studies cited above and on the findings of some authors of this paper to solve nonlinear hyperbolic equations in the context of atmospheric modeling (Marras et al., 2015, Section 5), we propose a parameter-free shock capturing scheme to detect the presence of spurious modes in the proximity of strong gradients. The model that we propose – we will often refer to it as *Dyn-SGS* to indicate its *Dynamic Sub-Grid Scale* nature – was first defined in Nazarov and Hoffman (2013) for the linear finite element solution of compressible flows with shock waves. It was applied to stabilize high-order Galerkin methods in the context of stratified, low Mach number atmospheric flows by some of the authors in Marras et al. (2015). It was recently used successfully to remove oscillations from the DG solution of nonlinear acoustic waves in Kelly et al. (2017). *Dyn-SGS* is based on the idea of scale splitting, where the flow scales are split into resolvable and unresolvable for a given computational grid. The unresolved scales are parameterized via the subgrid scale (SGS) model at hand. It must be borne in mind throughout the manuscript that *Dyn-SGS*, unlike the sub-grid scale models designed for LES that are built from physical reasoning, is merely a numerical tool meant to remove the spurious oscillations from the solution of nonlinear wave equations and does not have, a priori, a physical meaning. Among its characteristics, being parameter-free and dynamically adaptive as a function of the solution residuals are possibly the most attractive ones. Furthermore, this model is independent of the underlying numerical approximation, which makes it naturally applicable to CG and DG alike, as well as to finite elements, finite volumes, and finite differences.

2. Governing equations

Let $\Omega \in \mathbb{R}^d$ be a fixed domain of space dimension d with boundary Γ and Cartesian coordinates $\mathbf{x} = [x]$ in 1D and $\mathbf{x} = [x, y]$ in 2D; in both cases, we will use z to identify the direction of gravity which is orthogonal to $\mathbf{x}$ and points downward. Let $t \in \mathbb{R}^+$ identify time. Given Ω and t we define the velocity vector $\mathbf{u}(t, \mathbf{x})$ whose