Advances in Water Resources 113 (2018) 141–154

Contents lists available at ScienceDirect

Advances in Water Resources

journal homepage: www.elsevier.com/locate/advwatres

Shear and shearless Lagrangian structures in compound channels

Enrile F.*, Besio G., Stocchino A.

DICCA, Dipartimento di Ingegneria Civile, Chimica e Ambientale, Università degli Studi di Genova, Italia

A R T I C L E I N F O

Keywords:
Shear Lagrangian Structure
Shearless Lagrangian Structure
Ridges
Trenches
FTLE
Shearlines
River dynamics

A B S T R A C T

Transport processes in a physical model of a natural stream with a composite cross-section (compound channel) are investigated by means of a Lagrangian analysis based on nonlinear dynamical system theory. Two-dimensional free surface Eulerian experimental velocity fields of a uniform flow in a compound channel form the basis for the identification of the so-called Lagrangian Coherent Structures. Lagrangian structures are recognized as the key features that govern particle trajectories. We seek for two particular class of Lagrangian structures: Shear and shearless structures. The former are generated whenever the shear dominates the flow whereas the latter behave as jet-cores. These two type of structures are detected as ridges and trenches of the Finite-Time Lyapunov Exponents fields, respectively. Besides, shearlines computed applying the geodesic theory of transport barriers mark Shear Lagrangian Coherent Structures. So far, the detection of these structures in real experimental flows has not been deeply investigated. Indeed, the present results obtained in a wide range of the controlling parameters clearly show a different behaviour depending on the shallowness of the flow. Shear and Shearless Lagrangian Structures detected from laboratory experiments clearly appear as the flow develops in shallow conditions. The presence of these Lagrangian Structures tends to fade in deep flow conditions.

1. Introduction

Natural rivers and, quite often, artificial channels are characterized by cross-sections composed by a deeper main channel and shallower floodplains. For this reason they are usually referred as “compound channels”. Flows of these streams are defined as predominantly horizontal since their horizontal dimensions greatly exceed the vertical one (Jirka, 2001).

The analysis of mixing processes in natural streams is not a simple task as flow dynamics is strongly affected by channel irregularities. Flow velocity in the floodplains is lower than the one of the main channel, due to the water shallowness and to bed roughness typically higher than the main channel. As a result of the velocity gradient, shear occurs at the interface between the main channel and the floodplains. The presence of various Eulerian flow patterns most of which are characterized by large-scale vortical structures with vertical axes, i.e. macro-vortices, is well-known (Socolofksy and Jirka, 2004; Stocchino et al., 2011; Stocchino and Brocchini, 2010). The generation of these vortical structures can be described by two main approaches (Rowiński and Radecki-Pawlik, 2015): either as a shear instability at the junction of two streams (van Prooijen et al., 2005) or as an outcome of differential energy dissipation of shallow-water currents interacting with submerged obstacles (Soldini et al., 2004). The former approach casts an analogy between the transitional region of the compound channel and a free mixing layer. The latter identifies the driving mechanism for the generation and sustainment of the Eulerian macro-vortices in the vorticity generation owing to the depth jump across the cross-section. Stocchino and Brocchini (2010) showed that the shear layer thickness remains constant in compound channels. Such a condition is a peculiar consequence of the topographic forcing, i.e. the depth jump, generating the Eulerian macro-vortices. On the contrary, the shear layer generated by the junction of two streams on an even bottom tends to grow linearly. In order to clarify strengths and shortcomings of both, a detailed comparison between the approaches pursued by van Prooijen et al. (2005) and Soldini et al. (2004) should be carried out and the outcome of the numerical simulations compared. However, the issues raised by these two different approaches are not considered in the present work. Indeed, we aim to analyse experimental surface velocity fields under a Lagrangian perspective disregarding the Eulerian approach. Note that it is well-known that Eulerian and Lagrangian patterns do not always correspond (Haller, 2015).

An experimental investigation on the mixing processes, in terms of Lagrangian statistics of single and multiple particles, was presented by Stocchino et al. (2011). However, the role of flow inhomogeneity was disregarded in that study. This aspect is the main subject of the present work, where we aim to detect coherent patterns from Lagrangian measures in order to seek structures that characterise the compound channel. Key structures are located at the transition from the main

* Corresponding author.
E-mail address: francesco.enrile@edu.unige.it (E. F.).

https://doi.org/10.1016/j.advwatres.2018.01.006
Received 1 August 2017; Received in revised form 5 January 2018; Accepted 6 January 2018

channel to the lateral channels (floodplains) and approximately along the axis of the main channel. Therefore, we focus on Lagrangian structures that shape trajectory patterns.

The present analysis mainly relies on the computations of the Finite Time Lyapunov Exponents (FTLE) fields along with related trenches (Beron-Vera et al., 2010) and ridges (Shadden et al., 2005), as a first diagnostic tool. However, FTLE trenches and ridges are not always a signature of the presence of material lines. Despite such a shortcoming, they are still a valuable tool to understand the dynamics of the flow. In particular, ridges are able to reveal the regions of motion that are kinematically the most active (Allshouse and Peacock, 2015a). We then manage to isolate two types of heuristic structures that are mostly disregarded in previous studies: Jet-Cores (JC), i.e. shearless structures, and Shear Lagrangian Structures (SLS), respectively. JC were studied by Beron-Vera et al. (2010) and Farazmand et al. (2014). In the present work we apply the methodology detailed in the former study based on FTLE trenches. Besides, we characterize the behaviour of heuristic JC resulting from FTLE trenches by applying the methodology described by Allshouse and Peacock (2015b). The same method is also applied to ridges of FTLE fields that mark heuristic SLS. Such a conclusion is proven by testing heuristic SLS against their shear properties.

A further characterization of shear is carried out upon the rigorous definitions of Lagrangian Coherent Structures (LCS) (Haller, 2011; Haller and Beron-Vera, 2012). Among the general family of LCS, SLS are features dominated by a bulk shear typical of parallel flows. Herein, SLS are detected in order to mark the fundamental geometry of shear patterns. Note that SLS and JC are usually defined and studied on the basis of analytical velocity fields, whereas the main goal of the present study is to deeply investigate realistic flow conditions in a laboratory model of a typical river configuration. Heuristic SLS calculated as FTLE ridges and rigorous SLS calculated from the geodesic theory of transport barriers are compared and a nice agreement is found.

Summing up, experimental data of time-dependent, two-dimensional Eulerian velocity fields (Stocchino et al., 2011; Stocchino and Brocchini, 2010) are employed to calculate numerical trajectories upon which JC and SLS are estimated against their shear properties. Rigorous SLS are also calculated as shearlines that minimize their geodesic deviation.

The paper proceeds with Section 2 devoted to the definition and formulation of the LCS identification techniques. Then, in Section 3 we describe the velocity fields employed and we asses their two-dimensionality. Section 4 describes in details the LCS that can be detected in shallow, intermediate, and deep flow conditions. Finally, Section 5 is devoted to the conclusions and closes the paper.

2. Theoretical background

A fluid is usually studied applying the well-known results of continuum mechanics and we follow this approach. A fluid body $\mathscr{B}$ is made of elements called particles ξ. In order to describe the position of these particles we establish a one-to-one correspondence between the particles and the coordinates of a reference system, i.e. a triple of real numbers. We introduce Lagrangian coordinates $\boldsymbol{\xi} = (\xi^1, \xi^2, \xi^3)$ as a material coordinate system that label fluid particles. Since any two systems of coordinates are related by a continuously differentiable transformation we can introduce Eulerian coordinates as

$$\boldsymbol{x} = \boldsymbol{\Phi}(t; t_0, \boldsymbol{\xi}) \tag{1}$$

where $\boldsymbol{\Phi}$ is the flow map. The Eulerian coordinates denote the position of a point fixed in what can be called the laboratory frame (Thiffeault and Boozer, 2001). The transformation showed in equation (1) can be inverted in the neighbour of a point provided that the Jacobian exists and does not vanish (Aris, 1962).

The study of fluid flows cannot be carried out disregarding velocity fields. Indeed, velocity fields are the core of fluid mechanics and time-dependent velocity fields are generally written as $\boldsymbol{v}(\boldsymbol{x}, t)$. The trajectory of particles are curves solutions of

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}(\boldsymbol{x}, t) \tag{2}$$

with initial conditions $\boldsymbol{x}(t_0, \boldsymbol{\xi}) = \boldsymbol{\xi}$.

We can regard Eq. (2) as a set of ordinary differential equations and evaluate on a finite time interval $T = (t_1 - t_0)$ the distance that two initial close particles can experience. Therefore, if we consider as initial conditions $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_0 + \boldsymbol{\epsilon}$ we can evaluate the final distance between the two particles applying a linearisation (Allshouse and Peacock, 2015b):

$$\delta\boldsymbol{x}(t_1) = \boldsymbol{\Phi}(t_1; t_0, \boldsymbol{\xi}_0) - \boldsymbol{\Phi}(t_1; t_0, \boldsymbol{\xi}_0 + \boldsymbol{\epsilon}) \approx \nabla\boldsymbol{\Phi}(t_1; t_0, \boldsymbol{\xi}_0)\boldsymbol{\epsilon} \tag{3}$$

where $\nabla\boldsymbol{\Phi}(t_1; t_0, \boldsymbol{\xi}_0)$ is called the flow map gradient and it is a tensor represented by a matrix the entries of which are $\nabla\Phi^i_j = \partial x^i/\partial\xi^j$. We impose two restrictions on $\nabla\boldsymbol{\Phi}$. Firstly, an infinitesimal material element $d\boldsymbol{x}$ must not split along its evolution and coalescence of two material elements is not allowed. This is the physical interpretation of the condition on the Jacobian of Eq. (1). The second restriction imposes that the deformation must preserve orientation, i.e. three right-handed material elements $d\boldsymbol{x}$, $d\boldsymbol{y}$ and $d\boldsymbol{z}$ satisfying $d\boldsymbol{x}\wedge d\boldsymbol{y}\cdot d\boldsymbol{z} > 0$ are transformed into three material elements satisfying $d\boldsymbol{x}(t) \wedge d\boldsymbol{y}(t)\cdot d\boldsymbol{z}(t) = (\nabla\boldsymbol{\Phi}d\boldsymbol{x}) \wedge (\nabla\boldsymbol{\Phi}d\boldsymbol{y})\cdot(\nabla\boldsymbol{\Phi}d\boldsymbol{z}) = \det(\nabla\boldsymbol{\Phi})d\boldsymbol{x} \wedge d\boldsymbol{y}\cdot d\boldsymbol{z} > 0$. By writing $\nabla\boldsymbol{\Phi}d\boldsymbol{x}$ we denote the product between the matrix $\nabla\boldsymbol{\Phi}$ and the vector $d\boldsymbol{x}$, i.e. a contraction that results in a vector. Scalar product between vectors is indicated as $(\cdot)$. The second restriction implies that the Jacobian of Eq. (1) must satisfy the following condition:

$$J = \det(\nabla\boldsymbol{\Phi}) > 0 \tag{4}$$

The magnitude of the final distance can be evaluated as (Shadden et al., 2005):

$$\begin{aligned} |\delta\boldsymbol{x}(t_1)| &= \sqrt{\delta\boldsymbol{x}(t_1)\cdot\delta\boldsymbol{x}(t_1)} = \sqrt{[\nabla\boldsymbol{\Phi}\delta\boldsymbol{x}(t_0)]\cdot[\nabla\boldsymbol{\Phi}\delta\boldsymbol{x}(t_0)]} = \\ &= \sqrt{\delta\boldsymbol{x}(t_0)\cdot[\boldsymbol{C}\delta\boldsymbol{x}(t_0)]} = \sqrt{\boldsymbol{\epsilon}\cdot(\boldsymbol{C}\boldsymbol{\epsilon})} \end{aligned} \tag{5}$$

where $\boldsymbol{C}$ is the Cauchy-Green tensor defined as $\boldsymbol{C} = (\nabla\boldsymbol{\Phi})^T\nabla\boldsymbol{\Phi}$ where $(\cdot)^T$ denotes the transpose. It is possible to prove that matrix $\boldsymbol{C}$ is positive definite and symmetric. Since we analyse 2D velocity fields, $\boldsymbol{C}$ has two eigenvectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ associated with two eigenvalues $0 < \lambda_1 \leq \lambda_2$, respectively.

Maximum stretching occurs when $\delta\boldsymbol{x}(t_0)$ is chosen such that it is aligned with the eigenvector associated with the maximum eigenvalue of $\boldsymbol{C}$, i.e.:

$$\max|\delta\boldsymbol{x}(t_1)| = \sqrt{\lambda_2}\,|\overline{\delta\boldsymbol{x}}(t_0)| \tag{6}$$

where $\overline{(\cdot)}$ indicates alignment with the eigenvector associated with the maximum eigenvalue λ_2 of the Cauchy–Green tensor. Since $\delta\boldsymbol{x}(t_0) = \boldsymbol{\epsilon}$, Eq. (6) can be recast to obtain

$$\max|\delta\boldsymbol{x}(t_1)| = e^{\sigma_{t_0}^{t_1}|T|}|\bar{\boldsymbol{\epsilon}}| \tag{7}$$

where

$$\sigma_{t_0}^{t_1} = \frac{1}{|T|}\log\sqrt{\lambda_2} \tag{8}$$

represents the (maximum) Finite-Time Lyapunov Exponent (FTLE) calculated on a finite integration time T.

The eigenvectors of $\boldsymbol{C}$ define directions of initial separations for which neighbouring particles are converging or diverging. Since we are interested in the most active regions of the fluid flow from a kinematic point of view, we define the FTLE in Eq. (8) as a function of the maximum eigenvalue. Panel a) of Fig. 1 shows the deformation in the neighboured of a point under the action of the flow map. Computation of FTLE can be carried out in forward time, i.e. from t_0 to $t_0 + T$, or in backward time, i.e. from $t_0 + T$ to t_0. Identification and classification of the main features of these scalar fields is the subject of the next paragraphs.

Eckmann and Ruelle (1985) showed how λ_2 tends asymptotically to