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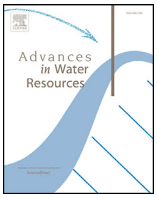

Stochastic longshore current dynamics

Juan M. Restrepo[a,*], Shankar Venkataramani[b]

[a] Department of Mathematics, and College of Earth, Ocean, and Atmospheric Sciences, Oregon State University, Corvallis, OR 97331, USA
[b] Department of Mathematics and Program in Applied Mathematics, University of Arizona, Tucson AZ 85721, USA

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ABSTRACT

We develop a stochastic parametrization, based on a 'simple' deterministic model for the dynamics of steady longshore currents, that produces ensembles that are statistically consistent with field observations of these currents. Unlike deterministic models, stochastic parameterization incorporates randomness and hence can only match the observations in a statistical sense. Unlike statistical emulators, in which the model is tuned to the statistical structure of the observation, stochastic parametrization are not directly tuned to match the statistics of the observations. Rather, stochastic parameterization combines deterministic, i.e physics based models with stochastic models for the "missing physics" to create hybrid models, that are stochastic, but yet can be used for making predictions, especially in the context of data assimilation.

We introduce a novel measure of the utility of stochastic models of complex processes, that we call *consistency of sensitivity*. A model with poor consistency of sensitivity requires a great deal of tuning of parameters and has a very narrow range of realistic parameters leading to outcomes consistent with a reasonable spectrum of physical outcomes. We apply this metric to our stochastic parametrization and show that, the loss of certainty inherent in model due to its stochastic nature is offset by the model's resulting consistency of sensitivity. In particular, the stochastic model still retains the forward sensitivity of the deterministic model and hence respects important structural/physical constraints, yet has a broader range of parameters capable of producing outcomes consistent with the field data used in evaluating the model. This leads to an expanded range of model applicability. We show, in the context of data assimilation, the stochastic parametrization of longshore currents achieves good results in capturing the statistics of observation *that were not used* in tuning the model.

1. Introduction

Longshore (alongshore) currents are ubiquitous oceanic flows in nearshore environments (see Lentz and Fewings, 2012, for a descriptive review). The two mechanisms responsible for their existence are wave stresses and alongshore sea elevation gradients. Because longshore currents affect nearshore bathymetry and beach morphology, and are responsible for a great deal of nearshore transport, models for these currents are of great practical utility.

Presently, deterministic wave-resolving models are used with good results, capturing some of the complex dynamics of the nearshore, including longshore currents (see Chen et al., 2003, Choi et al., 2015, Noyes et al., 2004, Noyes et al., 2005. See also Cienfuegos et al. (2010)). Of note are wave-resolving, depth-integrated models based upon the Boussinesq equations, such as funwaveC (Feddersen (2014)). These have shown considerable forecasting skill (See for example, Feddersen, 2007, and Suanda and Feddersen, 2015, in which field data on eddy variability and dye and drifter dispersion are compared to funwaveC model output). The success of funwaveC in capturing a wide range of nearshore oceanic phenomena rests upon its generality: it includes higher order dispersion (see Nwogu, 1993), a general bottom drag parametrization (see Feddersen, 2007), wave breaking momentum transformation parametrization via Newtonian damping (see Kennedy et al., 2000) and the breaking viscosity model (Lynett, 2006).

Non-wave resolving complex models of the nearshore ocean environment exist as well. These also have compared favorably with certain aspects of longshore current dynamics and observations, such as longshore shear instabilities. They can also capture other nearshore flows, such as rip currents, consistent with observations (Allen et al., 1996, Ozkan-Haller and Kirby, 1999, Uchiyama et al., 2009 and references contained therein).

None of the models capture longshore current observations in a statistically faithful manner. By fidelity we mean that the time series generated by the model and the observations are indistin-

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[*] Corresponding author.
E-mail address: restrepo@math.oregonstate.edu (J.M. Restrepo).

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guishable, statistically. The most familiar modeling approach to improving model fidelity is to resolve and include more physics (or better physics). The present state-of-the-art in longshore modeling is the wave-resolving models mentioned above. An approach that directly focuses on obtaining statistical fidelity is statistical emulation (see Currin et al., 1991). In this strategy observations are used to build phenomenological models. The fundamental modeling strategy consists of proposing a basic statistical distribution or a regression model. Structure is built into the model by calibrating the model's correlations and other statistical dependences with data.

The goal of this paper is to propose and demonstrate the use of an alternative modeling approach, called stochastic parametrization. It is an intermediate between the deterministic and the emulator approaches to modeling physical phenomena. This strategy yields a model that has as much structure and physics as possible, leaving as little as possible to chance. A good stochastic parametrization yields structure in the statistics of its time series by the blend of deterministic and stochastic elements, rather than by tuning the model using data to incorporate the correlations and structure of the observations.

A common use for stochastic parametrization is to incorporate in a computationally efficient way unresolved dynamics that cannot be ignored even if the model focused on large spatio-temporal scale phenomena. A familiar practical example of deterministically parametrizing small scales is via homogeneization (see Bensoussan et al., 1978): when the small scales offer a certain amount of scale separation and it is statistically homogeneous, the small scale averages that persist appear in the resulting deterministic model as complementary or added terms. A more sophisticated approach, in computational fluid mechanics, is large eddy simulations (LES) of turbulent flows. In that case the complementary/added terms themselves have their own dynamics which come from closures of higher moment statistics. Stochastic parametrization is meant to increase a model's fidelity, but unlike LES, it will not do so rationally. For the longshore current model featured in this paper the stochastic parametrization will be used to capture the small scale variability present in the observations, enhancing this way its fidelity, not its rationality.

In this study we will purposely choose the simplest possible model for longshore current dynamics, a balance model, as a starting point. Clearly, a model that already has improved physics would be a better choice for the development of an operational stochastic model, but a simple and familiar model makes it plain, to what extent the stochastic parametrization is effective in enhancing the original deterministic model's fidelity. Balance models for longshore currents capture nothing more than the most basic of physical processes, albeit under strong assumptions. Nearly all of the physics in longshore current models are captured by empirical parametrization: the models incorporate parametrized wave radiation (or the vortex force), wave breaking, turbulence, stress and drag forces. The longshore model we will use is derived from the vortex force formulation for the evolution of waves and currents in the nearshore (see McWilliams et al., 2004, and Lane et al., 2007). The vortex force model is a general, non-wave resolving, model. It was used in Weir et al. (2011) to describe the evolution of rip currents, as well as longshore currents in Uchiyama et al. (2009). This model (see Appendix A) will be referred to as the *vortex force model*, in order to distinguish it from the simpler and specialized *balance model* for longshore current dynamics.

The plan of the rest this paper is as follows: after describing essential background to the nature of the data, in Section 2, we introduce in Section 3 the longshore balance model that will be used as a basis for the stochastic parametrization. Section 3 discusses the conditions under which the longshore balance model is derived from the non wave-resolving, wave-current interaction, vortex force model. Stochastic parametrization can lead to better *consistency of model sensitivity*, at the expense of increased uncertainty. The topic of consistency of sensitivity will be taken up in Section 4. A model that has consistent sensitivity will have a wide range of physically-meaningful parameter combinations with which to capture a broad spectrum of physical outcomes. The balance model will be used to explore and illustrate the consequences of sensitive consistency. The stochastic parametrization, inspired by the data and constrained by the physics of longshore currents, is introduced in Section 5. Stochastic parametrization of unresolved physics, as evidenced by the data, is used to construct a stochastic balance longshore model. Suggesting a simple model for observational data that is clearly non-Gaussian will lead us to introduce Gaussian mixtures. With this choice of stochastic parametrization the stochastic longshore model is shown to compare favorably with observations. Notably, the model captures correlations present in the data without having to explicitly put these into the model. Furthermore, the stochastic longshore model will be shown to have consistent sensitivity. However, the use of Gaussian mixtures reduces the fidelity of the model. The point of using the mixture model will nevertheless make the model amenable to simple linear/Gaussian data assimilation methods. Data assimilation is a very useful methodology for combining observations and models. Stochastic models are well suited for this application, as will be shown in Section 6, using the proposed stochastic longshore model and observations in a concrete data assimilation example calculation.

2. Longshore current observational data

In the process of constructing a stochastic parametrization, as well as in testing the resulting stochastic model, we will make use of field observations. We will use the data set collected in Duck, North Carolina by Herbers, Elgar, Guza, and Birkemeier, Long and Hathaway (see Elgar et al., 1994). Henceforth, we shall refer to this data as the *Duck data*. The Duck data repository provides data, and in particular, information on nearshore flow mean velocity. It also has recordings of ocean pressure, temperature, and depth, over the course of several months. It also contains information on the peak frequency, direction, and sea elevation amplitude of incoming waves. Bathymetry as well as the conditions under which the data was obtained is available as well. The Duck data as well as ancillary information are available from:

frf.usace.army.mil/pub/Experiments/DUCK94/SPUV.

A plot of a typical bottom topography cross section appears recreated in Fig. 1. The plot also shows the data collection devices and data collection locations. The cross shore and longshore components of the velocity were collected at a sampling rate of 2 Hz for 10,784 s, every 3 h. The data collection spanned the months of August, September, October, and early November, 1994. Sporadic instrumentation failure lead to interruptions in data collection. The cross-shore velocity component is zero on average for most of the data gathering campaign, and is ignored in this study. The specific "SPUV data" streams we will make use of are from measurement locations v12, v13, v14, v15, which were located approximately at the offshore coordinates 205, 220, 240, and 260 m, respectively. The transect we will work with is at roughly 930 m in the alongshore (y) direction. At these stations the data exhibits good signal to noise characteristics. The locations correspond a location right before the waves shoal and break. We will also be using another set of data, collected during the same time period as the SPUV longshore current data, and it consists of wave elevation, wave period, and wave direction, further out from shore, roughly 900 m offshore, where the water depth is approximately 8m. This data is available from frf.usace.army.mil/pub/Experiments/DUCK94/FRF. Fig. 2 shows a more general view of the bathymetry, reconstructed