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Verification and refinement of a two species Fish Wars model

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ABSTRACT

In this paper, we analyse a modification of the well known two species Fish Wars model of Fischer and Mirman—a dynamic game which models players catching two species of fish—Their model is modified in order to encompass both the possibility of depleting resources and the Allee effect. In addition, we verify the results of Fischer and Mirman regarding equilibria in such games, which, to the best of our knowledge, have not yet been proven.

1. Introduction

"The tragedy of the commons", which describes a wide range of phenomena arising in, among other things, the exploitation of public or interdependent renewable resources by a number of players—is a very important present-day problem. Competition between the players may lead to the extinction of a species or the depletion of resources, while even imperfect cooperation may result in its preservation. This is crucial in the case of marine resources. The tragedy of the commons was introduced in the seminal paper of Hardin (1968) and first modelled as a dynamic game by Levhari and Mirman (1980). This model was an attempt to quantitatively describe the phenomena resulting in Fish Wars, like the Cod War between Iceland and Great Britain, which was stated to be the main motivation of their paper.

In this introduction, we concentrate only on one branch of dynamic games, applications to the exploitation of public or interdependent renewable resources, which is a direct continuation of the seminal paper of Levhari and Mirman (1980), to which the model of Fischer and Mirman (1992) belongs. Readers interested in other papers on the game theoretic modelling of the exploitation of such resources are referred to the reviews e.g. by Long (2011, 2012) or monograph series e.g. Carraro and Filar (1995).

The model of Levhari and Mirman (1980) (LM for short) uses a logarithmic payoff function to emphasize the fact that the depletion of a resource or the extinction of a species is disastrous for a society whose subsistence is based on fishing. In addition, to make the dynamics of the availability of the resource realistic, it is assumed that when there is no human interference there exists a positive stable steady state of the biomass, which can be determined, together with exponential growth. Linear growth is also considered. An infinite time horizon is considered. The model of Levhari and Mirman has been developed in many ways. Okuguchi (1981) generalizes it to n players. Mazalov and Rettieva (2010b, 2009) additionally consider natural mortality and concentrate mainly on cooperation, with the possibility of asymmetry in Rettieva (2014). Fischer and Mirman (1992, 1996) generalize the LM model to two species with various interdependencies. Mazalov and Rettieva (2010a) and Rettieva (2012) consider a variant of Fischer and Mirman's model that reflects the facts that fishing grounds have different locations and fish can migrate (see also further research by those authors). Doyen et al. (2016) consider n species and ask questions about *the tragedy of the commons* and the preservation of biodiversity. Wiszniewska-Matyszkiel (2005) considers linear dynamics and allows subsets of the set of players to make decisions together. In Wiszniewska-Matyszkiel (2014a), she also analyses asymmetric players. Kwon (2006) introduces partial cooperation. Breton and Keoula (2012) introduce cooperation and examine the stability of coalitions in the case of delays in information, while in Breton and Keoula (2014), they consider asymmetric players. Koulovatianos (2015) introduces randomness into the growth function of the resource and assumes that players learn about the state of the resource. Antoniadou et al. (2013) introduce randomness of a known form into the growth function and calculate Markov-perfect Nash equilibria using the Bellman equation. Dutta and

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Sundaram (1993) examine a wider class of problems, defining "the tragedy of the commons" as overexploitation of a resource above the "golden rule" level, and look for solutions which satisfy the "golden rule". Wiszniewska-Matyszkiel (2016, 2014b) compares Nash equilibria to belief-distorted Nash equilibria—where players do not exactly know their influence on a resource. Therefore, players have beliefs, which are not necessarily consistent with the actual dynamics, and respond to their beliefs in an optimal way, which results in the verification of these beliefs. Hanneson (1997) modifies the game to an infinitely repeated supergame and investigates the possibility of enforcing cooperative behaviour. The payoff functions contain a logarithmic component. Cave (1987) considers an environmental agreement based on the LM model, according to which countries first calculate a Pareto optimal profile and threaten to immediately return to the Nash equilibrium strategy in the case of the other player defecting. They state that this problem is equivalent to one in which strategies are based on a different, history-dependent information structure. Amir and Nannerup (2006) develop the analysis of various information structures in the LM model. Wiszniewska-Matyszkiel (2008) considers a model with continuous time.

One development that is especially important from the point of view presented in this paper was introduced by Fischer and Mirman (1992) (throughout this paper we shall use the abbreviation FM). They extend the model to one with two species, where each of the two players (firms or countries) catches one of these species. This is the first model to analyse the noncooperative exploitation of interdependent species by two different decision makers. This leads to a very complicated problem (the corresponding problem of dynamic optimization by a single user was considered later by Mesterton-Gibbons, 1996). A wide range of interdependencies are considered: symbiosis, prey-predator and competition for a common prey, which lead to interesting conclusions. Although the results are widely cited and regarded as being correct, to the best of our knowledge, they have not yet been proven. So, formally, they still remain a hypothesis.

Obviously, in the case of a logarithmic payoff function and linear or exponential resource regeneration, the traditional sufficient condition, based on the Bellman equation and a terminal condition at infinity, does not hold (see e.g. Bellman, 1957, Blackwell, 1965, Stokey et al., 1989 or any textbook on discrete time dynamic optimization, e.g. Judd, 1998, Feinberg and Shwartz, 2002 or dynamic games, e.g. Başar and Olsder, 1982). Moreover, small modifications of this model, without any substantial change in its logic, which do not guarantee that the biomass not only cannot be depleted, but even cannot converge to zero, do not improve the situation (see e.g. the example in Wiszniewska-Matyszkiel, 2011).

Until recently, there have been no tools to derive a complete proof regarding equilibria for a model similar to the one presented by FM or the other game theoretic papers cited above. To fill in this gap, Wiszniewska-Matyszkiel (2011), proves a generalized version of the traditional sufficient condition, with a relaxed terminal condition, which can be used to prove optimality in linear-logarithmic and exponential-logarithmic dynamic optimization problems. Its application to dynamic games has already appeared in Wiszniewska-Matyszkiel (2014a). This paper shows that the application of this approach to FM works in the case of two species with symbiosis (or one species with multiple fishing grounds). Therefore, in this case we can complete the proofs of the results of FM and verify that the strategy profile proposed therein is really a Nash equilibrium. In addition, some other previous results, including the results of Mazalov and Rettieva (2010a) and Rettieva (2012) can be proven to be correct, since the terminal condition is fulfilled.

In the remaining cases—the prey-predator model and two species competing for a common prey—the solutions proposed by FM cannot be verified using this approach without changing the dynamics of the ecosystem or constraining the set of strategies. Moreover, the value functions proposed by FM are undefined when species become extinct. In fact, the model excludes the possibility of a species becoming extinct, which does not seem realistic. In addition, in the case of one species being close to extinction when there is interspecies competition for food, or the predator being close to extinction in the case of the prey-predator model, according to the original FM model, the biomass of the other species tends to infinity, which is also unrealistic.

The same problem appears in the generalization of FM made by Doyen et al. (2016) to many species. Here, the terminal condition is even less tractable than in FM and the same objections concerning modelling can be pointed out. It has to be emphasized again, that unless the terminal condition for a dynamic optimization problem with unbounded payoffs is shown to be satisfied, then any results based on the sufficiency of the Bellman equation cannot be treated as correct. On the other hand, there are very few papers with even an incomplete analysis of games with interactions between at least two species, which is crucial whenever we use the word "ecosystem". Therefore, such an analysis is really needed. Moreover, according to the authors' knowledge, this paper contains the first completely proven results regarding equilibria in an infinite horizon dynamic Fish War game with more than one species. Recently, several counterexamples have been found, even for simple dynamic optimization problems (an extremely simple counterexample is given in Singh and Wiszniewska-Matyszkiel, 2017), showing that not checking the terminal condition can lead to the derivation of false value functions and false optima and/or equilibria in the case of unbounded payoffs.

Another problem, not mentioned in the Fish Wars stream of papers, is the Allee effect (introduced by Joosten, 2016 in games involving the extraction of resources), which states that below some critical level of biomass, a species starts to degenerate and soon becomes extinct.

To meet the criteria of providing complete proofs of results and modelling the behaviour of an ecosystem to reflect the Allee effect, such an analysis is performed.

The paper is composed as follows. Section 2 presents the original FM model and results that can be proven in the case of the original model or a slight modification. Section 3 is devoted to a modification introducing the possibility of depletion and the Allee effect. Both of these sections state the main results, while, in order to facilitate reading, all the technicalities are presented in Appendices. The main tool used for analysis is presented in Appendix A, while the proofs of the results are in Appendices B.1–B.3.

2. The Fischer–Mirman model and its verification

We consider a common fishery exploited by two players. Those players may represent firms, in some cases also countries. The game is dynamic, with discrete time and the infinite horizon.

There are two species, therefore, the set of state variables, denoting the biomasses of both species, is $\mathbb{X} = \mathbb{R}_+^2$.

However, like in FM, we assume that each of the players can exploit only one species: player i can extract only species i. In our formulation, the decision of a player represents the catch rate. For player i, it is denoted by c_i. The set of i's possible decisions is $[0, 1]$, since the player cannot extract more fish than available. In some cases, we have to exclude 1, since otherwise the next stage biomass for the other species may be undefined. We return to this problem in the sequel.

We consider so called feedback strategies (in some papers on dynamic optimization or dynamic games, they are called closed loop strategies), $C_i: \mathbb{X} \to [0, 1]$ (or $C_i: \mathbb{X} \to [0, 1)$ whenever we have to restrict decisions to $[0, 1)$). The set of such strategies of player i is denoted by $\mathbb{C}_i$, while $\mathbb{C} = \mathbb{C}_1 \times \mathbb{C}_2$ is the set of all strategy profiles. For transparency, we use notation c_i for a decision, i.e., a single value of the catch rate, which is a number, while C_i for a whole strategy (a function).

Emphasizing the form of strategies is important, since in dynamic games, unlike in dynamic optimization problems, considering different types of strategies usually results in different, not equivalent, equilibria (for discussion and rare cases of equivalence see Wiszniewska-