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Assessing flow resistance in gravel bed channels by dimensional analysis and self-similarity

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ABSTRACT

In this paper a new flow resistance equation for open channel flow, based on the integration of a power velocity profile, was tested for gravel bed channels. First this flow resistance equation, theoretically deduced by dimensional analysis and incomplete self-similarity condition, was reported. Then a relationship between the Γ function of the velocity profile, the channel slope and the Froude number was calibrated by the available laboratory measurements of flow velocity, water depth and bed slope carried out in 416 flume experimental runs with a gravel bed. Then the relationship for estimating Γ function and the theoretical resistance equation was tested by 83 independent flume measurements. The analysis also showed that the proposed flow resistance equation allows an estimate of the Darcy-Weisbach friction factor which is more reliable and accurate than that obtained by a semi-logarithmic flow resistance law or a variable-power resistance equation, calibrated with the same gravel bed measurements. For testing the applicability of the proposed Γ function (Eq. (17)), whose coefficients were estimated by flume measurements, available fields measurements were used. The analysis demonstrated that a scale factor (equal to 0.7611) is necessary to convert Γ values obtained by flume measurements into those corresponding to gravel bed rivers. The similitude between flow resistance in a gravel bed flume and in a gravel bed river is governed by the Γ function and a scale factor, equal to 1.6, is required to upscale the Darcy-Weisbach friction factor values obtained by flume measurements to the river case. In conclusion, the analysis showed that the Darcy-Weisbach friction factor for gravel bed channels can be accurately estimated by the proposed theoretical approach based on a power-velocity profile.

1. Introduction

Bathurst (1982) stated that a channel has a cobble and boulder bed when the median size d_{50} of its bed particles is $>$ 64 mm, the effects of vegetation are negligible and it is characterized by a *transition* or *large-scale* hydraulic condition which occurs for a ratio between the uniform flow depth h and the bed diameter d_{84}, for which 84% of the bed particles are finer, less than or equal to 4 (Bathurst et al., 1981; Colosimo et al., 1988; Ferro, 1999; Reid and Hickin, 2008).

In principle, the theoretical deduction of the flow-resistance law can be obtained by integration of a known flow-velocity distribution in the cross-section (Ferro, 1997, 2003a, 2003b; Powell, 2014). This deduction continues to be one of the main challenges for uniform open channel flow hydraulics and the available theoretical results refer to defined boundary conditions (fixed bed) and some simple cross-section shapes (circular and rectangular very wide) since in these cases the velocity profile is known (Ferro and Pecoraro, 2000).

For a *small scale roughness*-condition, occurring when the uniform flow depth is much higher that the characteristic size of the particle arranged on the channel boundary ($h/d_{50} > 20$ according to Bray (1987) or $h/d_{84} > 4$), and a two-dimensional open channel flow, in the fully turbulent part of the inner region and in the outer region (Coleman and Alonso, 1983; Kirkgóz, 1989; Ferro and Pecoraro, 2000; Ferro, 2003a, 2003b) the velocity profile is described by a logarithmic distribution. Ferro and Baiamonte (1994), using the velocity profiles measured in a gravel-bed flume, established that the logarithmic velocity profile fits well the velocity measurements for a relative depth, which is equal to the ratio between the distance from the bottom y and h, less than or equal to y_{max}/h being y_{max} the distance from the bottom where the maximum velocity occurs.

For a *small scale roughness*-condition, the integration of the logarithmic velocity profile yields to a semi-logarithmic flow resistance law (Ferro, 2003a):

$$\sqrt{\frac{8}{f}} = A_o + B_o \log\left(\frac{R}{k_s}\right) \tag{1}$$

in which f is the Darcy-Weisbach friction factor, A_o and B_o are two coefficients, R is the hydraulic radius and k_s is the roughness height.

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For flow with depth sediment ratio h/d_{84} in the range 1–4, as initially observed by Marchand et al. (1984), the velocity profile is S-shaped (Bathurst, 1988; Ferro and Baiamonte, 1994) with near-surface velocities marked higher than those near-beds. For a large-scale roughness condition, the near bed local conditions affect the shape of the velocity profile, which could not have a regular shape and its theoretical deduction presents some difficulties. Ferro and Pecoraro (2000) for the two conditions of small- and large-scale roughness applied the incomplete self-similarity theory to establish the velocity profile in a gravel be channel. The deduced power velocity distribution was able to reproduce experimental velocity profile for which the maximum velocity occurs at the free surface.

For a *large-scale roughness condition* the flow resistance is affected by the shape, the arrangement and the concentration of coarser elements (Bathurst, 1978; Bray, 1982; Lawrence, 1997). Ferro (1999) carried out an experimental investigation, using a ground layer on which a number N of boulders were arranged, varying the boulder concentration from 0 to 83%. For transition and large-scale roughness, Ferro (1999) empirically established the following flow resistance law:

$$\sqrt{\frac{8}{f}} = b_o + b_1 \log\left(\frac{h}{d_{84}}\right) \tag{2}$$

in which b_o and b_1 are coefficients. The intercept b_o becomes constant, and equal to – 1.5, for boulder concentration values > 50%. This last result states that for concentration values > 50% a quasi-smooth (skimming) flow occurs (Morris, 1959). Using data pairs (h/d_{84}, $\sqrt{8/f}$) corresponding to boulder concentration ranging from 0 to 44%, Ferro and Giordano (1991) estimated $b_o = 1.4084$ and $b_1 = 7.8468$.

The integration of the velocity distribution in the cross-section carried out by Ferro and Pecoraro (2000) confirmed the applicability of a semi-logarithmic equation, like Eq. (2), for all investigated bed shapes of the gravel bed flume.

Bathurst (2002) highlighted that existing flow resistance equations for large-scale and transition roughness conditions ($h/d_{84} < 10$) have a significant empirical content, being derived by fitting to an ensemble of data for different sites, and may be in error by typically 30%. Bathurst (2002), using a dataset characterized by slopes in the range 0.2–4% and $h/d_{84} < 11$, concluded that relative submergence based on d_{84} should be an excellent primary predictor of the Darcy-Weisbach friction factor and the dependence of $\sqrt{8/f}$ on h/d_{84} is more accurately described by a power law than a semi-logarithmic law as Eqs. (1) and (2).

Rickenmann and Recking (2011), using a wide data set of field measurements, tested the ability of several flow resistance equations and concluded that the best overall performance is obtained by the Ferguson (2007) approach which combines two power law flow resistance equations that are different for deep (small-scale roughness condition) and shallow flows (large-scale roughness condition).

The Manning-Strickler (MS) equation, which is currently applied for deep rivers with lower slopes (small-scale roughness condition), implies a 1/6 power relationship between $\sqrt{8/f}$ and h/d_{84} and semi-logarithmic equations (like Eqs. (1) and (2)) which can be approximated by a power law over a limited range of h/k_s using specific exponents. However several authors, using different calibration data sets, proposed the generalized power law

$$\sqrt{\frac{8}{f}} = a\left(\frac{h}{k_s}\right)^b \tag{3}$$

in which a and b are numerical coefficients whose estimate depends on the used calibration dataset. Lawrence (1997; 2001) and Nikora et al. (2001), for *large-scale roughness condition* ($1 \leq h/k_s < 4$), concluded that the roughness elements affect all water depths in the flow and proposed that flow resistance is mainly due to form drag on roughness elements (Roughness-Layer resistance, RL). For the shallow flow condition, Lawrence (1997, 2000) proposed a mixing-length model (mixing length scales with k_s) which implies a linear resistance relation between $\sqrt{8/f}$ and h/k_s.

Ferguson (2007) deduced a variable-power resistance equation (VPE) which admits as end members of a range of possible power law resistance equations, corresponding to different hydraulic conditions, the following MS friction law (deep flows) and the RL relation (shallow flows), respectively:

$$\sqrt{\frac{8}{f}} = a_1\left(\frac{h}{k_s}\right)^{1/6} \qquad \text{(for deep flows)} \tag{4a}$$

$$\sqrt{\frac{8}{f}} = a_2\left(\frac{h}{k_s}\right) \qquad \text{(for shallow flows)} \tag{4b}$$

in which a_1 and a_2 are two constants. The constant a_1 ranges from 7 to 8, and according to Parker (1991) assumes a value equal to 7.3 when d_{84} is used as roughness scale, the constant a_2 varies from 1 to 4 (Ferguson, 2007).

The synthesis proposed by Ferguson (2007) is the following VPE that is asymptotic to the MS and RL equations as h/d_{84} becomes very large or very small, respectively:

$$f = 8\left[\frac{\left(\frac{h}{d_{84}}\right)^{1/3}}{a_1^2} + \frac{\left(\frac{h}{d_{84}}\right)^{2}}{a_2^2}\right] \tag{5}$$

Ferguson (2007) tested Eq. (5) by a compilation of measured velocities in natural streams with different channel morphologies, slopes ranging from 0.07 to 21% and 0.1 $R/d_{84} < 26$, and concluded the VPE, which performs as well as any existing resistance law, may be a useful tool for predicting flow velocity by a single equation over a wide range of conditions.

The difficulties and uncertainties due to the integration of the velocity distribution in a cross-section justify the circumstance that the Chezy, the Manning and the Darcy-Weisbach uniform flow resistance equations continue to be the most commonly applied empirical formulas (Powell, 2014):

$$V = C\sqrt{R\ i} = \frac{i^{1/2}\ R^{2/3}}{n} = \sqrt{\frac{8\ g\ R\ i}{f}} \tag{6}$$

in which V is the cross-section average velocity, C is the Chezy coefficient ($m^{1/2}\,s^{-1}$), n is the Manning coefficient ($m^{-1/3}\,s$), i is the channel slope and g is acceleration due to gravity.

Further advances in understanding the flow resistance require a carefully study of the involved processes and the availability of laboratory and field data for testing both the empirically derived and theoretical flow resistance equations.

In this paper the dimensional analysis and the self-similarity theory are used to theoretically establish the flow-resistance law in a gravel bed channel. In particular, the incomplete self-similarity hypothesis is applied to theoretically deduce the flow velocity profile which is integrated for obtaining the flow resistance law.

Then the theoretically deduced flow resistance law is calibrated by laboratory measurements of discharge, water depth and bed slope carried out in 416 experimental runs by Ferro and Giordano (1991) for a condition of large scale roughness ($0.88 \leq h/d_{84} \leq 4.14$) and with different values of boulder concentration.

In particular, the Γ function of the velocity profile, which was calibrated in previous papers (Ferro, 2017; Ferro and Porto, 2018) using field measurements characterized by $0.11 \leq i \leq 7.5\%$, $0.18 \leq F \leq 1.25$, and $0.95 \leq h/d_{84} \leq 6.83$, is recalibrated in this paper by using flume measurements corresponding to channel slope i ranging from 0.69 to 9.4%, $0.19 \leq F \leq 0.97$, a large scale roughness condition and boulder concentration less than or equal to 44%.

Then the theoretical flow resistance law is also verified by 83 independent flume measurements, carried out by Ferro and Baiamonte (1994), Baiamonte et al. (1995) and Ferro and Pecoraro (2000), for a