Geoderma 334 (2019) 15–23

Contents lists available at ScienceDirect

Geoderma

journal homepage: www.elsevier.com/locate/geoderma

Joint multifractal analysis for three variables: Characterizing the effect of topography and soil texture on soil water storage

Asim Biswas

School of Environmental Sciences, University of Guelph, Guelph, Ontario N1G 2W1, Canada

ARTICLE INFO

Handling Editor: Morgan Cristine L.S.

Keywords:
Power law
Fractal theory
Scaling
Partition function
Soil water

ABSTRACT

Multifractal analysis describes heterogeneity in the distribution of a variable by characterizing and summarizing the variability across scales. Joint multifractal analysis has been widely employed to characterize scale relationships between two variables co-existing along a single geometric support. In this study, joint multifractal analysis was used for three variables coexisting in the same geometric support to describe the influence of topography (relative elevation) and soil texture (sand content) on water storage within a soil profile. Soil water storage, as well as sand content and relative elevation, were measured down to 1.4 m depth along a 576 m long transect in the hummocky landscape of central Saskatchewan, Canada. Joint multifractal analysis was conducted to consider both the strange attractor formalism and the method of moments. The variability in soil water storage, sand content and relative elevation was scale dependent and showed multifractal behavior. The spatial variability in relative elevation was strongly reflected on water storage across the analyzed spatial scales but the joint multifractal spectrum for sand content and water storage suggested a lower degree of correlation. The change in multifractality was also observed when there was high variability in relative elevation and texture. This clearly demonstrated the capability of joint multifractal analysis to completely characterize the scaling behavior among three variables.

1. Introduction

Information on spatial and temporal distribution of soil water is a key input in monitoring soil water balance and the global hydrological cycle, assessing land-atmospheric interactions, understanding a large number of surface and subsurface hydrological processes, testing the performance of various engineered covers, and validation of climate and hydrological models (Koster et al., 2004; Rodriguez-Iturbe et al., 1999; Sivapalan, 1992; Western et al., 2002). Many physical factors (e.g. topography, soil properties) and environmental processes (e.g. rainfall, evapotranspiration, runoff, and snow melt) operating at different intensities over a variety of scales (Entin et al., 2000; Goovaerts, 1998) give rise to complex and nested patterns or spatial (temporal) organization (Western et al., 1999) in soil water variability as a function of spatial (temporal) scales (Biswas and Si, 2011c; Gomez-Plaza et al., 2000; Kachanoski and Dejong, 1988). Basically, the spatial variability of soil water is the reflection of the spatial variability of hydrological processes. Thus, information on soil water variability at the scale of measurement provides indication on the underlying soil hydrological processes at that scale (Banerjee et al., 2011; Goovaerts, 1998). However, as the variability in soil water is controlled by several factors and processes operating at multiple scales, the multi-scale nature of hydrological processes should be characterized and quantified to better understand the hydrological cycle (Biswas and Si, 2011b; Biswas and Si, 2011c; Gomez-Plaza et al., 2000). For example, at small catchment and hill slope scale, factors like water routing processes (Beven and Germann, 1982; Dunne and Black, 1970; Moore et al., 1988), differential radiation effects (Moore et al., 1993; Western et al., 1999), heterogeneity in soil (Famiglietti et al., 1998; Hu et al., 1997; Seyfried, 1998) and vegetation (Hupet and Vanclooster, 2002; Qiu et al., 2001) affect soil water content/storage patterns on and within the landscape. In contrast, atmospheric, geologic, and climatic variability determine the organization of soil water over large area (Brocca et al., 2007; Entin et al., 2000; Schneider et al., 2008; Seyfried, 1998). The contribution of certain processes, though dominant at one scale, may have weaker control at other scales and relatively smaller contribution towards overall hydrological dynamics. Now at a scale, the effect from weaker processes can be masked by other dominant processes (Biswas and Si, 2011a; Ji et al., 2016; Kachanoski and Dejong, 1988). Multi-scale variability of soil water and thus hydrological processes make hydrological studies challenging and managing decisions difficult at a scale other than the scale of measurement. Therefore, characterizing variations at the scale of measurement and transferring information from one scale to another, also known as scaling, is critical

E-mail address: biswas@uoguelph.ca.

https://doi.org/10.1016/j.geoderma.2018.07.035
Received 6 October 2017; Received in revised form 18 July 2018; Accepted 27 July 2018
0016-7061/

for understanding complex hydrological dynamics.

Semivariogram, the central to geostatistical analysis and a second-order statistical moment of a spatial variable (Journel and Huijbregts, 2003), has been used to characterize the spatial pattern in soil water (Brocca et al., 2007; Entin et al., 2000; Western et al., 2002). The underlying assumption of this is the 'intrinsic hypothesis' where all the regionalized variables are considered Gaussian and it only depends on the separation distance or lag distance between samples (stationary process) (Journel and Huijbregts, 2003). Nevertheless, it can effectively reveal the spatial distribution and autocorrelation of soil water within the study area. However, the complex behavior of soil water variability is created by both irregularity and structure for different length scales resulted in several closely-spaced and nested scales of variations, occurring over short distances (Burrough, 1983a; Goovaerts, 1997). To overcome the inadequacy of a single 'range' to account for abrupt changes of the mean of the target soil property e.g. soil water storage and the lack of deterministic similarity of variations at different scales, nested semivariogram model can be calculated (Burrough, 1983b). In this model, several non-overlapping and independent random functions each with its own weight (based on dominance and contribution) and scale were used to represent the underlying soil processes determining variability of soil water. However, practical application of this model requires a priori knowledge on the dominant scales and magnitudes of the processes controlling the pattern of soil water, which is rarely available (Zeleke and Si, 2006). Moreover, having quantified these relationships at multiple scales, it is required to transfer information from one scale to another for developing connection among 'process scale', 'measurement scale' and 'management scale' (Biswas and Si, 2011d) but limited by its capability. Additionally, traditional correlation analysis, commonly used to identify the dominant factors of soil water storage (Biswas et al., 2012; Vachaud et al., 1985), can quantify the relationship only at the scale of measurement and needs to be analyzed across scales.

The scaling of a soil property e.g. soil water is possible if the distribution of some statistical parameters (e.g. variance) within a geographical support space or spatial domain remains the same at other geographical support spaces or spatial domains (Cheng, 1999). This means that the feature in the distribution of soil water will not change if the spatial domain is multiplied/divided by a common factor, also known as scale-invariance or self-similarity (Hu et al., 1997; Kim and Barros, 2002). Therefore, the probability of measuring a value will vary inversely as a power of that value and is known as power function, a typical scaling process. As the spatial distribution of soil water is known to follow the power law function (Hu et al., 1997; Ji et al., 2016; Kim and Barros, 2002; Mascaro et al., 2010), spatial variability can be investigated and characterized using fractal theory (Mandelbrot, 1982). As the soil water storage could be a response of complex nonlinear processes acting over a variety of scales, the spatial distribution characterization and quantification requires multiple scaling indices (multifractal scaling). Therefore, multifractal scaling indices can provide insights into the interrelationships between systems and the organization about the underlying mechanism (Cheng, 1999; Martin-Sotoca et al., 2018). For example, Ji et al. (2016) and Kim and Barros (2002) observed a multifractal behavior of surface soil water as a result of the temporal evolution of wetting and drying in a semi-arid and humid environment, respectively. Similarly, Mascaro et al. (2010) reported the multifractal behavior of soil water and used it to develop downscaling models for remote sensing measurements.

As the scaling properties of the spatial patterns of soil water storage change with scales (Brocca et al., 2010; Ji et al., 2016; Kim and Barros, 2002), it is necessary to understand the relationship with controlling factors at different scales. If the multifractal behavior of soil water storage co-exist with other controlling factors in the same spatial domain, the relationship can be jointly analyzed using the joint multifractal analysis (Meneveau et al., 1990). Joint multifractal analysis has been used to characterize the interaction between crop yield and terrain indices (Kravchenko et al., 2000; Zeleke and Si, 2004), grassland productivity and terrain indices (Banerjee et al., 2011), volume and number based soil particle size distribution (Li et al., 2011), and soil water retention parameters and soil texture (Wang et al., 2011). However, to date, joint multifractal analysis has been used to explore the relationship between only two variables at different scales. The scaling relationship between soil water storage and individual factors may not explore the relationship sufficiently as the overall effect from multiple factors could be highly nonlinear and non-additive (Biswas and Si, 2011b). For example, among other factors, soil water storage is known to be controlled by topography (Brocca et al., 2010; Western et al., 1999), soil texture (Cosh et al., 2008; Vachaud et al., 1985) or a combination (Biswas et al., 2012; Tallon and Si, 2004). In this situation, characterizing the joint variability between either soil water storage and topography or soil water storage and soil texture at multiple scales may not reveal the complete picture. Extending the joint multifractal analysis (Meneveau et al., 1990) to multiple variables (Pavon-Dominguez et al., 2015) could provide a complete picture of soil water storage spatial variations as controlled by different factors at different scales. Therefore, the objective of this study was to extend the joint multifractal analysis for three variables to characterize the effect of topography (e.g. relative elevation) and soil texture (e.g. sand) on soil water storage at multiple scales. A 3D visualization of joint multifractal spectrum was used to describe the effect of multiple variables at multiple scales.

2. Materials and methods

2.1. Study site and data collection

The data (soil water storage, relative elevation and sand) used in this study were collected along a north-south direction transect of 128 points separated by 4.5 m regular intervals. The transect was established in 2004 within the St. Denis National Wildlife Area (52°12′N lat. and 106°50′W long.) located in central Saskatchewan, Canada as part of a bigger project to study soil water dynamics in the Prairie Pothole Region. Several other publications have used part of the dataset collected over the years to answer various questions regarding soil water dynamics. More detailed information on the study site, actual data set, data collection procedures and publications using the dataset can be found in Biswas et al. (2012). Briefly, the landscape of the study area is mainly hummocky with a complex sequence of slopes (10 to 15%) extending from different sized rounded knolls to depressions. The depressions, also known as potholes, were formed from buried ice chunks during the last deglaciation (Huel, 2000) and almost act as an individual micro-watershed. This is a landscape characteristic of the North American Prairie pothole region covering approximately 780,000 km^2 area from the north-central United States to south-central Canada (National Wetlands Working Group, 1997) and it is very important for its unique hydrological and ecological functions. The soil of the study area is mainly Dark Brown Chernozem developed from moderately fine to finely textured, moderately calcareous glacial till (Saskatchewan Centre for Soil Research, 1989). The climate is semi-arid with long-term annual average precipitation of 360 mm and an annual average air temperature of 2 °C (AES, 1997). The vegetation of the study area is mixed grass.

The surface soil water (0–20 cm) was measured using a time domain reflectometry (TDR) probe and a metallic cable tester (Model 1502B, Tektronix, Beaverton, OR, USA). Sub-surface water content was measured at every 20-cm depth down to 140-cm using a neutron probe (Model CPN 501 DR Depthprobe, CPN International Inc., Martinez, CA, USA). A site-specific calibration was used for the neutron probe (Biswas et al., 2012) and a standard calibration equation (Topp and Reynolds, 1998) was used for the TDR measurements. Soil water storage (SWS) was calculated from soil water content and depth of each layer and added together to get the total SWS in the profile (0–140 cm). In this