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Research papers

Semi-analytical solution for pressure transient analysis of a hydraulically fractured vertical well in a bounded dual-porosity reservoir

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ABSTRACT

We study the role of a hydraulic fracture on the pressure transient behavior of a vertical well producing from a bounded (or finite) dual-porosity formation. A combination of Laplace transform (LT) and the finite Fourier cosine transform (FFCT) are used to solve the diffusivity equation. The presented analysis allows identification of five flow regimes, including: 1) early linear flow, 2) volumetric depletion of natural fractures, 3) natural-fracture radial flow, 4) transition from natural-fracture radial flow to total (natural fractures and matrix) radial flow, and 5) pseudo-steady state flow. The results reveal that the interporosity flow coefficient, storativity ratio, natural-fracture permeability anisotropy, and reservoir size play significant roles on the identified flow regimes compared to the hydraulically fractured well location and reservoir shape. The developed solution can be useful for well test analysis by generating a new set of type curves or can be applicable to a forward model for estimating parameters of reservoir. This study presents a new semi-analytical solution which finds application in well testing of hydraulically fractured wells in dual-porosity formations.

1. Introduction

Pressure transient behavior of fractured formations has been widely studied in hydrological sciences and petroleum engineering (Warren and Root, 1963; Kazemi, 1969; de Swaan, 1976; Moench, 1984; Zimmerman et al., 1993; Hamm and Bidaux, 1996; De Smedt, 2011; Dejam et al., 2013). The well pressure response in dual-porosity reservoirs was first described by Warren and Root (1963). A complete review of related works in literature can be found elsewhere (Hassanzadeh et al., 2009). Due to the very low mobility of stored fluids in tight matrix blocks of a naturally fracture formation, it is sometimes required to apply advanced techniques (such as hydraulic fracturing) for increasing mobility of fluid in these reservoirs and enabling flow to the production wells. Therefore, a better understanding of concepts of fluid flow in hydraulically fractured formations for improving fluid extraction is one of the important areas of research. Hydraulic fracturing is an effective and operative stimulation method and has been extensively implemented. In this technique, fluid (which consists primarily of water but also include a variety of chemical additives) is injected at a rate adequate to increase the wellbore pressure that can exceed the fracture pressure of the reservoir rock. The stress induced by the pressure generates hydraulic fractures that increase the formation permeability and cause larger flow rate of fluids into the production well (Stuart, 2012). In other words, hydraulic fracturing is a well-stimulation technique in which reservoir rock is fractured by a pressurized liquid. In fact, the purpose of this technique is to enhance hydrocarbon production from wells with damaged zones or from reservoirs with low permeabilities.

Traditionally, well test analysis has been used as a tool in the determination of formation properties (Raghavan et al., 1972; Gringarten et al., 1975; Cinco-Ley et al., 1978; Cinco-Ley and Samaniego, 1981). During the last decades, numerous efforts have been performed for modeling of the pressure transient behavior for vertical wells with or without hydraulic fractures in single- and dual-porosity reservoirs. All these efforts were performed based on the application of the source/sink term combined with a mathematical method (including Green function, Laplace transform, etc.) to solve unsteady-state pressure diffusivity equation.

First, Russell and Truitt (1964) presented data for pressure drawdown for a vertically fractured well in the center of a closed square by solving the diffusivity equation with a source term using a finite difference approximation. Later, Raghavan et al. (1972) applied a technique to interpret the pressure transient behavior of a vertically fractured well under buildup testing. Their results substantiate prior results

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Nomenclature

a	coordinate of hydraulic-fracture left end (L)
a_D	dimensionless coordinate of hydraulic-fracture left end
b	coordinate of hydraulic-fracture right end (L)
b_D	dimensionless coordinate of hydraulic-fracture right end
c	well coordinate (L)
c_D	dimensionless well coordinate
c_f	fracture compressibility (LT^2/M)
c_m	matrix compressibility (LT^2/M)
e	exponential function
f	a function of s
h	thickness of dual-porosity formation and hydraulic fracture (L)
h_D	dimensionless thickness of dual-porosity formation and hydraulic fracture
k_{fD}	natural-fracture permeability anisotropy in the x-y or horizontal plane
k_{fx}	natural-fracture permeability in the x-direction (L^2)
k_{fy}	natural-fracture permeability in the y-direction (L^2)
k_m	matrix permeability (L^2)
l	length of dual-porosity reservoir (L)
l_D	dimensionless length of dual-porosity reservoir
m	variable of finite Fourier cosine transform (FFCT) with respect to x_D
n	variable of FFCT with respect to y_D
p_f	fluid pressure in natural fractures (M/LT^2)
p_{fD}	dimensionless fluid pressure in natural fractures
P_{fD}	dimensionless natural-fracture pressure in Laplace domain
$\overline{P}_{fD}$	FFCT of P_{fD} with respect to x_D
$\widetilde{\overline{P}}_{fD}$	FFCT of $\overline{P}_{fD}$ with respect to yD
p_i	initial fluid pressure in dual-porosity reservoir (M/LT^2)
p_m	fluid pressure in matrix (M/LT^2)
p_{mD}	dimensionless fluid pressure in matrix
P_{mD}	dimensionless matrix pressure in Laplace domain
p_{wD}	dimensionless hydraulically fractured well pressure
P_{wD}	dimensionless hydraulically fractured well pressure in Laplace domain
q	flow rate (L^3/T)
s	variable of Laplace transform
t	time (T)
t_D	dimensionless time
w	width of dual-porosity reservoir (L)
w_D	dimensionless width of dual-porosity reservoir
x	horizontal coordinate (L)
x_D	dimensionless horizontal coordinate
x_f	hydraulic-fracture half-length (L)
y	vertical coordinate (L)
y_D	dimensionless vertical coordinate

Greek letters

cos	cosine
H	Heaviside step function
ln	natural logarithm
ℓ_{t_D}	operator for LT with respect to t_D
ℓ_s^{-1}	operator for inverse LT with respect to s
sin	sine
$\wp_c^{x_D}$	operator for FFCT with respect to x_D
$(\wp_c^m)^{-1}$	operator for inverse FFCT with respect to m
$\wp_c^{y_D}$	operator for FFCT with respect to y_D
$(\wp_c^n)^{-1}$	operator for inverse FFCT with respect to n
ϕ_f	natural-fracture porosity
ϕ_m	matrix porosity
λ	interporosity flow coefficient
λ_m	eigenvalues for FFCT with respect to x_D
λ_n	eigenvalues for FFCT with respect to y_D
σ	shape factor ($1/L^2$)
δ	Dirac delta function (1/L)
μ	fluid viscosity (M/LT)
ω	storativity ratio
∞	infinity

Subscripts

c	FFCT
D	dimensionless
f	fracture
i	initial
m	matrix
m	variable of FFCT with respect to x_D
n	variable of FFCT with respect to y_D
s	variable of LT
t_D	dimensionless time
w	well

Superscripts

m	variable of FFCT with respect to x_D
n	variable of FFCT with respect to y_D
x_D	dimensionless horizontal coordinate
y_D	dimensionless vertical coordinate
$-$	FFCT with respect to x_D
$\sim$	FFCT with respect to y_D
-1	inverse

of Russell and Truitt (1964).

Then, Gringarten et al. (1974) presented data for drawdown for an infinite-conductivity vertical fracture, which is located at the center of a closed square drainage region and producing a slightly compressible fluid at a constant rate. Subsequently, Gringarten et al. (1975) applied their previous results from (Gringarten et al., 1974) to interpret the well test data from the vertical fractured wells.

Thereafter, Cinco-Ley (1974) and Cinco-Ley et al. (1975) derived an analytical expression for the pressure transient behavior for an inclined fracture accompanied with a vertical well by implementation of the instantaneous point-source solution. Next, Raghavan et al. (1978) were the first developed an analytical model that evaluates the impact of the fracture height on the pressure transient behavior of a vertical fracture. They derived their model using the Green function product solution technique, which was presented by Gringarten and Ramey (1973).

Later, Cinco-Ley et al. (1978) developed a mathematical model to study the pressure transient behavior of a well with a finite-conductivity vertical fracture, which is located in an infinite slab reservoir by applying Green and source functions and the Newman product technique. Then, Cinco-Ley and Samaniego (1981) presented a new method to analyze pressure data for a well intersected by a finite-conductivity vertical fracture. They developed their technique applying the bilinear flow theory, which considers linear flow not only in fracture but also in formation. They coupled the flow equations in the fracture and formation and solved those using successive applications of the Laplace transforms with respect to time and vertical coordinate.

Subsequently, Rodriguez et al. (1984a,b) developed semi-analytical solutions to investigate the influence of the partial penetration of infinite and finite conductivity fractures on the pressure transient behavior of a vertically fractured well. Thereafter, Ozkan and Raghavan