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Research papers

An implicit friction source term treatment for overland flow simulation using shallow water flow model

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ABSTRACT

Aiming at resolving the numerical problems caused by the improper friction source term treatment when simulating overland flow using 2D shallow water flow models, a proposed implicit method computing the friction source term is developed in this work. The method is able to not only accurately evaluate the tricky thin overland flow by considering the flow velocity varying in a single time step, but also eliminates the redundant iterations for regular implicit approach through converting the implicit scheme to an explicit equation. It is therefore an improved one in terms of accuracy and efficiency comparing to the existing methods. Furthermore, it is an independent part and can be straightforwardly and universally incorporated into any FVM based shallow water flow model. Such performances are validated against three test cases involving theoretical and practical overland flow problems. The results prove the proposed method treating friction source term could simulate overland flow problems in a relatively more accurate and efficient way, and therefore can effectively help extend the applicability of the shallow water models reliably computing both the open channel and overland flows.

1. Introduction

Numerical models for surface water flow, including overland and open channel flows, play an essential role in solving hydrological, hydraulic and environmental problems, for example, evaluating surface runoff (Unami et al., 2009; Costabile et al., 2013; Simons et al., 2014) and flood routine (Cook et al., 2009; Huang et al., 2013), providing flow field for the contaminant transport and erosion models (Heng et al., 2009; Mügler et al., 2011; Jomaa et al., 2013; Kim et al., 2014; González-Sanchis et al., 2015; Zhao et al., 2018). Traditionally, hydrological models or simplified hydrodynamic models (Lighthill and Whitham.,1955; Govindaraju.,1988) are usually used for overland flow simulations at a catchment scale. Among the commonly used surface water flow models, the dynamic wave models based on the Saint-Venant equations, or shallow water equations (SWE), are generally considered to be the most physically based one due to their capabilities of computing flow properties in time and space (Borah, 2011; Yeh et al., 2011; Costabile et al., 2012; Kim et al., 2013; Xia et al., 2017). However, some challenges remain for the development of efficient and robust Godunov-type SWEs models to the real-world overland flow applications, because of the complex overland flow patterns over complex domain topography.

One of the major challenges related to solving the SWEs for overland flows is the treatment of friction source terms. The friction source terms are usually expressed as a non-linear function of the flow velocity and depth, e.g. the Manning formula. One inherent property of the SWEs with such friction source terms is that the behavior of the equation changes dramatically as the water depth becomes small, which is likely to cause numerical problems and is demonstrated herein by analyzing the momentum equation in 1D SWEs as:

$$\frac{\partial q}{\partial t} + \frac{\partial qu + 0.5gh^2}{\partial x} = S_s + S_f \tag{1}$$

where q denotes the unit-width discharge, t is the time, x is the spatial coordinate, u is the depth-averaged velocity in the x- direction, h is the water depth, g is the gravity acceleration, S_s and S_f are the slope source term and the friction source term which are represented by ghi and $-C_f u\ |u|$, respectively, i denotes the bed slope and C_f is the roughness

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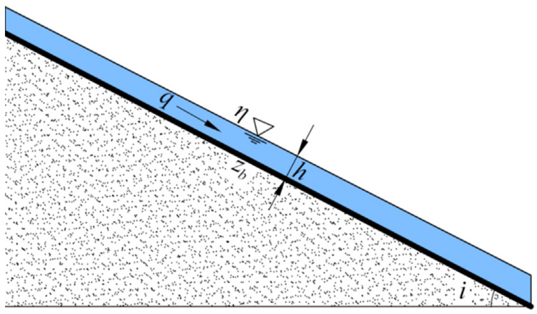

Fig. 1. Thin water flow over a slope.

coefficient evaluated by $C_f = gn^2/h^{1/3}$ where n is the Manning coefficient. As plotted in Fig. 1, for a thin water flow over a slope with the initial water depth h of 0.001 m, and the initial unit-width discharge of 0.0025 m^2/s, the updated q at the time level of $n + 1$ will be computed after a small time step using the explicit method. Because of the thin flow property, the scale of the velocities is much large than that of water depth (Costabile et al., 2013), therefore, the advection and pressure terms are assumed to be neglected. The friction source term and slope source term therefore become dominant due to the scale of squared velocities. Using the explicit method, the flow variables are assumed as those of at the current state when computing those at the next time level. Within a single time step, the friction source term computed may be much higher than that of the slope source term, some value even high enough to reverse the flow as shown in Table 1, which is reflected by the sign change between the two time levels. It is clear that the phenomena will not occur physically. The physical process is that if the friction force is lower than total driving force, the flow will be accelerated and the friction will subsequently increase until an equilibrium is achieved; otherwise, the flow will be decelerated to develop a lower velocity and the friction will then decrease until reaching a new equilibrium.

Obviously, the cause of the problem is the drastically varying velocity within a time step that cannot be well represented by using explicit approach. The similar problem has been addressed by introducing an '*asymptotic behavior*' of the SWEs in the presence of vanishing water depth (Jin, 2012). As the water depth becomes small, the relaxation time to the above-mentioned equilibrium state may be much smaller than the time step determined by the Courant-Friedrichs-Lewy (CFL) condition. But using a time step bigger than the relaxation time may lead to numerical instability, causing the so-called '*stiffness problem*' (Xia et al., 2017). Therefore, for accurate and stable simulation of overland flows, an SWE model must include a proper friction term discretization method to remove the 'stiffness'.

To resolve this problem, an explicit numerical scheme must use very small time steps to maintain numerical stability under such a situation. As shown in Table 1, a Courant number reduced to 0.005 could preserve a relatively reliable result. Xia et al. (2017) also noticed that the local equilibrium between friction and bed slope can be reached much faster than the time step determined by the CFL condition in practical overland flow simulations involving small water depth. The use of CFL condition for explicit scheme will therefore lead to prohibitive computational cost. Another approach as that suggested in (Liang et al., 2009; Hou et al., 2013a, Hou et al., 2014) is to constrain the friction to evade the flow reversion. This approach just assumes the local equilibrium is the flow turning up to be still, and therefore is not based on a general case for overland flow. Several implicit schemes have been developed to cope with the "stiff" friction source terms and preserve numerical stability, e.g., (Brufau et al., 2004; Costanzo and Macchione, 2006; Liang et al., 2007, Costabile et al., 2013), in which θ denotes the implicitness degree of the friction term discretization. The implicitness degree may vary according to the flow property to achieve a satisfactory result and impede its application. In (Xia et al., 2017), the full implicit method is applied straightly to solve the friction source term in the momentum equation rather than reformulate into an explicit form to ensure the flow velocities relaxed to the correct equilibrium state in a single time step when the friction becomes predominantly large. However, the iterations integrated in an explicit scheme solving the flux and slope source terms will inevitably increase the computation burden.

In this paper, a new approach is developed based on an implicit concept to deal with the friction source terms of the SWEs for overland flow simulation. It can effectively relax the flow toward the equilibrium state in a single time step and can be easily integrated into a commonly used explicit scheme, and therefore is more straightforward for implementation. The rest of the paper is organized as follows: Section 2 introduces the governing equations and the model framework; Section 3 presents the details of the proposed method for friction source term treatment; the behavior of the proposed method asymptotic behavior the flow to the equilibrium state is depicted in Section 4; the resulting overland flow model is firstly validated against the analytical tilted V-catchment test and then applied to reproduce a flood event at the Wangmaogou catchment in Section 5; finally, brief conclusions are drawn in section 6.

2. Numerical schemes

2.1. Governing equation

The shallow water equations (SWEs) are derived by assuming the hydrostatic pressure distribution and by depth integrating the equations of conservation of mass and momentum. In a vector form, a conservation law of the 2D shallow water equations can be written as:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} + \frac{\partial \mathbf{g}}{\partial y} = \mathbf{S} \tag{2}$$

$$\mathbf{q} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix},\ \mathbf{f} = \begin{bmatrix} q_x \\ uq_x + gh^2/2 \\ uq_y \end{bmatrix}, \mathbf{g} = \begin{bmatrix} q_y \\ vq_x \\ vq_y + gh^2/2 \end{bmatrix}, \mathbf{S} = S_b + S_f$$
$$= \begin{bmatrix} i \\ -gh\partial z_b/\partial x \\ -gh\partial z_b/\partial y \end{bmatrix} + \begin{bmatrix} i \\ -C_f u\sqrt{u^2+v^2} \\ -C_f v\sqrt{u^2+v^2} \end{bmatrix} \tag{3}$$

Table 1
Thin overland flow computation using hydrodynamic method.

q^n	h^n	n	C_f	u^n	S_f	dt	Courant No.	i	S_s	q^{n+1}
0.00025	0.001	0.01	0.00981	0.3	2.45E−03	4	0.5	0.025	9.81E−04	−0.00122
0.00025	0.001	0.01	0.00981	0.3	2.45E−03	4	0.5	0.01	3.92E−04	−0.00181
0.00025	0.001	0.01	0.00981	0.3	2.45E−03	4	0.5	0.005	1.96E−04	−0.00201
0.00025	0.001	0.01	0.00981	0.25	6.13E−04	1	0.125	0.025	2.45E−04	−0.00012
0.00025	0.001	0.01	0.00981	0.25	6.13E−04	1	0.125	0.01	9.81E−05	−0.00027
0.00025	0.001	0.01	0.00981	0.25	6.13E−04	1	0.125	0.005	4.91E−05	−0.00031
0.00025	0.001	0.01	0.00981	0.3	2.45E−05	0.04	0.005	0.025	9.81E−06	0.00024
0.00025	0.001	0.01	0.00981	0.3	2.45E−05	0.04	0.005	0.01	3.92E−06	0.00023
0.00025	0.001	0.01	0.00981	0.3	2.45E−05	0.04	0.005	0.005	1.96E−06	0.00023