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Full-field to sector modeling for efficient flow simulation in karst aquifers

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ABSTRACT

Brinkman's model is one of the common mathematical formulations used in modeling fluid flow in karst aquifers. The Brinkman's model is a hybrid formulation that allows the use of a single transport equation to model fluid flow in both the free-flow and porous regions of a karst aquifer, by transitioning between the Stokes equation and the Darcy's law. However, the Brinkman's model is computationally expensive, requiring the solution of four different equations to obtain the pressures and velocities in a three-dimensional aquifer. Darcy's model provides a cheaper alternative, because it allows the substitution of the Darcy's equation into the mass conservation equation and thus requires the solution of only one parabolic equation. However, while the Darcy's model produces accurate results within the porous medium, it fails to provide satisfactory results in the free-flow regions of the karst aquifers. We propose a sector modeling approach to model fluid flow in karst aquifers. The sector modeling approach takes advantage of both the computational inexpensiveness of the Darcy's model and the accuracy of Brinkman's model in the caves. This method was compared to the Brinkman's model and the Darcy's model. Three examples are presented to study the effectiveness of the sector modeling technique. The first example is a simple model consisting of a straight conduit surrounded by porous regimes on either side while the second and third examples are more complicated structures consisting of complex geometrical caves embedded in a highly heterogeneous porous medium. Results show that the sector modeling approach provides an excellent match to the Brinkman's model and with much faster computations. Specifically, sector modeling was 4.6 times faster than the Brinkman's model in the first example, 13 times faster in the second; and 25 times faster in the third example. We also observed negligible deterioration in accuracy of results from sector modeling when the size of the extracted sector relative to the full-field is reduced.

1. Introduction

Karst aquifers cover 12% of the terrestrial land and provide a source of drinking water to almost a quarter of the world's population (Ford and Williams, 2007; Andreo et al., 2010; Hartmann et al., 2014). Therefore, it is essential to accurately model the flow of water within these aquifers. The uniqueness of karst reservoirs is that the karst topography is characterized by the presence of highly porous and permeable caverns and channels formed by karstification (Jackson, 1997). The karst caves provide pathways that facilitate the movement of fluids and contaminants within such aquifers (Huntoon, 1999). Advection, diffusion and adsorption are the main physical processes responsible for the transport of contaminants, solutes and tracers, with advection being the primary process responsible for movement of these substances in caves (Field and Pinsky, 2000). Several numerical studies have been conducted on modeling the transport of chemicals within karst aquifers (Göppert and Goldscheider, 2007; Kincaid et al., 2002; Maloszewski et al., 1999; Morales-Juberías et al., 1997; Rivard and Delay, 2004).

Numerical modelling of groundwater flow within karst aquifers is challenging. The complexity in modelling arises due to the presence of non-Darcian flow elements in the transport equations, mainly due to the presence of fractures, vugs, and caves having varying sizes, connectivity and distribution depending on the depositional environment and the diagenetic processes involved. The presence of these geomorphological features introduces two major uncertainties which adversely affect the accuracy of the model: 1) the lack of knowledge of the exact position of the interface between porous medium and vugs/caves complicates the accurate modelling of flow in such features (Popov et al., 2009); and 2) uncertainties related to the physical properties of the different geomorphological features such as their sizes, connectivities, and distributions, directly affect the rate and direction of fluid flow in the aquifer (Kossack and Gurpinar, 2001). Different approaches have been developed to model the flow of groundwater in karst aquifers. The simplest is the lumped parameter model or the black box model (Scanlon et al., 2003; Wanakule and Anaya, 1993; Zhang et al., 1996).

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The main advantage of this model is that it requires limited information to produce results as it neglects the spatial dimensions of the aquifer. However, this omission of spatial dimensions is undesirable because it leads to a lack of information on the direction and rates of groundwater flow.

The other method of modelling flow within karst aquifers is the distributed parameter model which requires the discretization of the whole system into grid blocks. Thus, this method uses information on the spatial variability of the system. Generally, modeling fluid flow using the distributed parameter model involves combining the continuity equation with an appropriate transport equation. The continuity equation ensures that all mass flows in and out of the flow domain are accounted for. The transport equation describes the manner in which fluid mass is moved from one point to the other within the flow domain. Under the distributed parameter model, many approaches are currently used to model flow of fluid in karstic aquifers. One of these methods is the equivalent porous media (EPM) approach in which the fractures, vugs, caves, and the porous matrix are treated as one single equivalent porous medium with a single hydraulic conductivity (Ghasemizadeh et al., 2015; Long et al., 1982; Pankow et al., 1986; Scanlon et al., 2003). The disadvantage of this method is that it oversimplifies the complex flow patterns and may not be accurate. Another limitation of the EPM model is that it cannot accurately predict the direction of the flow.

The discontinuum approach (also known as Darcy-Stokes model) uses the Navier-Stokes equation to model flow in the free-flow region and the Darcy's equation in the porous region (Arbogast and Brunson, 2007; Arbogast and Gomez, 2009; Arbogast and Lehr, 2006; Peng et al., 2009). A slip velocity was introduced at the interface between the two regions using appropriate boundary conditions (Beavers et al., 1970; Beavers and Joseph, 1967). The Darcy-Stokes equation along with the interface condition models a slip velocity at the interface between the cave and the porous media. This yields a sharp change to Darcy's velocity in the porous region. This is not accurate because as the fluid transitions from the free-flow region to the porous media, the fluid velocity should gradually decreases inside the porous media until it becomes equal to the Darcy velocity (Neale and Nader, 1974).

Brinkman's equation (Brinkman, 1949) models flow in karst aquifers using a single equation that yields a simplified Navier-Stokes equation in the caves and breaks down into the Darcy's equation in the porous region. Hence, in the Brinkman's model, the whole system is treated as a single continuous domain (Yao and Huang, 2016). Several successful studies have been carried out to simulate the flow of single phase fluids in fractured, vuggy, microkarstic aquifers and reservoirs (Bi et al., 2009; He et al., 2015; Krotkiewski et al., 2011; Ligaarden et al., 2010; Mohamed et al., 2015; Popov et al., 2009, 2007b).

The Brinkman's model attempts to model flow within karst aquifers by simultaneously solving the mass continuity equation and the momentum equation (Brinkman's equation) for the pressures and velocity distribution within the karst aquifer. Because the Brinkman's model solves one continuity equation and up to three momentum balance equation per grid block, the method is very expensive particularly for large aquifers. The Darcy's model offers a cheaper alternative to the Brinkman's model because it does not require the solution of multiple equations per grid block. A simplification is made by replacing the velocity term in the mass continuity equation by the Darcy's equation. This allows solving for only the pressure distribution at the new time step in the aquifer and then computing the velocities using these pressures. While the Darcy's model gives reliable estimates of pressure and velocity distributions in the porous media, it gives an inaccurate velocity profile in the caves (Field, 1997).

We propose a sector modeling approach for fluid flow in Karst aquifers. This approach consists of three stages at every simulation time-step. The first stage involves solving the cheap Darcy's model on the entire aquifer (full-field model) while the second stage involves solving the more expensive Brinkman's model on much smaller sectors extracted from the reservoir (sector models). The third stage involves updating the result from the full-field model with those from the sector models. Thus, the sector modeling approach takes advantage of both the computational inexpensiveness of the Darcy's model and the accuracy of Brinkman's model in the caves. To avoid confusion, we define some terminologies introduced in this paper. Cave is synonymous with free-flow region while full-field is synonymous with entire aquifer. A full-field model involves solving the Darcy's model on the entire aquifer. A sector consists of a cave and a small porous region surrounding it. A sector is extracted from the full-field. A sector model involves solving the Brinkman's model on a sector while imposing dynamic flux boundary conditions (obtained from the full-field model) on the sector.

In addition to fluid flow modeling, tracer transport in the karst aquifer is implemented to enable us to compare the performance of sector modeling with those of the Darcy's and Brinkman's models. Three examples are presented to show the effectiveness of this method. The first example involves a simple aquifer model consisting of a cave surrounded on two sides by porous media. The second and third examples are more complex heterogeneous geological structures consisting of more realistic caves with randomly placed wells. The results from the sector modeling approach were compared with results from the Darcy's and Brinkman's models. The results show excellent match between the sector modeling approach and the Brinkman's model. In addition, the sector modeling approach was much faster than the Brinkman's approach.

2. Mathematical models of flow

This section describes the differential equations governing the flow of a single-phase fluid in a complex karst aquifer.

2.1. The Brinkman's model

Brinkman (1949) developed an equation that could model the coupled flow in an aquifer consisting of free-flow and porous regions. The Brinkman's equation is given by

$$\nabla p + \mu \overline{\overline{K}}^{-1}\vec{u} - \mu_{eff}\nabla^2\vec{u} = 0 \quad (1)$$

In Eq. (1), p is the pressure ($ML^{-1}T^{-2}$), μ is the fluid viscosity ($ML^{-1}T^{-1}$), $\overline{\overline{K}}$ is the permeability tensor (L^2), $\vec{u}$ is the velocity vector (LT^{-1}), and μ_{eff} is the effective fluid viscosity, a pseudo-parameter (introduced by Brinkman) whose value changes depending on the region of interest. An advantage of using the Brinkman's equation is that it yields a simplified form of Navier-Stokes equation in the free-flow region and the Darcy's equation in the porous media. This is achieved by adjusting the values of $\overline{\overline{K}}$ and μ_{eff}. When $\mu_{eff} = \mu$ and $\overline{\overline{K}} \to \infty$, Eq. (1) becomes the Stokes equation:

$$\nabla p - \mu\nabla^2\vec{u} = 0 \quad (2)$$

The Stokes equation in Eq. (2), is a simplification of Navier-Stokes equation when steady-state flow is assumed (Bird et al., 2002). Eq. (2) is used to model viscous flow in the free flow region consisting of vugs, fractures and caves. When $\mu_{eff} = 0$ and $\overline{\overline{K}} \ll \infty$ ($\overline{\overline{K}}$ is much less than ∞), Eq. (1) becomes Darcy's equation:

$$\nabla p + \mu \overline{\overline{K}}^{-1}\vec{u} = 0 \quad (3)$$

which is used to model flow of fluids in porous media (Darcy, 1856). When $\mu_{eff} = \mu$ and $\overline{\overline{K}} \ll \infty$ ($\overline{\overline{K}}$ is much less than ∞), Eq. (1) is used to model the Brinkman's region which starts just at the interface between the two regions and extends some distance into the porous region until the effect of the viscous shear term becomes negligible. Several studies have been conducted to find an appropriate value of the effective viscosity parameter in the transition zone (Adler, 1979; Belhaj et al., 2003; Durlofsky and Brady, 1987; Happel and Brenner, 1981; Howells,