Journal of Hydrology 564 (2018) 1110–1119

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Robust fractional programming approach for improving agricultural water-use efficiency under uncertainty

Qian Tan[a,*], Tianyuan Zhang[a,b]

[a] College of Water Resources and Civil Engineering, China Agricultural University, Beijing 100083, China
[b] State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu 610065, China

ARTICLE INFO

This manuscript was handled by G. Syme, Editor-in-Chief, with the assistance of Xaodong Zhang, Associate Editor

Keywords:
Agricultural water management
Fractional programming
Robust optimization
Uncertainty
Resource use efficiency
Watershed management

ABSTRACT

Water-use efficiency and uncertainty treatment are foci in the modeling of agricultural water management systems. To address these challenging issues, a robust fractional programming (RFP) method that coupled fractional programming with robust optimization was developed in this study to improve agricultural water-use efficiency under uncertainty. RFP improved upon the fractional programming by being able to tackle highly uncertain information without known distributions. It also extended the capability of the robust optimization method in addressing ratio optimal problems. To demonstrate its effectiveness and applicability, RFP was applied to a long-term agricultural water resources management problem in arid north-west China, where water scarcity and low water-use efficiency hindered local development. It generated benefit- and risk-explicit plans for crop pattern adjustments. Vegetables were recommended as the preferred crop. A number of scenarios combining different fluctuation and protection levels were analyzed and interpreted with practical implications. It was observed that higher water-use efficiency could be achieved through reducing parametric uncertainty and risk-aversion levels. Simulation experiments validated that the benefits claimed by the RFP model were sufficiently conservative and could be reliably achieved. The comparisons of RFP results against the baseline operations and those from two other alternatives demonstrated that, RFP could result in higher resource-use efficiency and controllable system-violation risks. The developed approach is also applicable to other optimization problems aiming at enhancing resource-use efficiency under uncertainty.

1. Introduction

Water scarcity is getting increasingly grim around the world with the growing needs of food production and economic development, especially in arid and semi-arid areas. Across all industries, agriculture is the largest consumer of water resources, taking up about 70% of total water consumption (Kang, 2017). Agricultural water pressure will further aggravate due to the increasing food demand and the booming population (Dai and Li, 2013; Cai et al., 2009). Enhancing irrigation water-use efficiency is necessary, especially in water-limited regions (Amini Fasakhodi et al., 2010).

Predecessors made great efforts to explore optimal schemes for allocating available surface water and groundwater resources in irrigated agriculture (Smith et al., 2000). Plenty of mathematical programming models were developed to deal with various specific problems and deemed effective (Cai et al., 2018). Particularly, multi-objective models providing compromised solutions were extensively applied (Singh, 2012; Park and Aral, 2004). For example, Wang et al. (2012) established a multi-objective model for optimizing a water-saving crop planting structure. Su et al. (2014) developed a multi-objective model to improve the efficiency of agricultural water-use with increased utilization proportion of green water. Allam et al. (2016) developed two multi-objective models for supporting sustainable reuse of drainage in irrigation. Tan et al. (2017) developed a multi-objective fuzzy robust programming method for supporting the optimal allocation of agricultural water and land resources. Although multi-objective programming methods could balance conflicting objectives (e.g. economic benefits, ecological benefits or water consumption) and coordinate the interests of different stakeholders, they often encountered difficulties in objectively weighting multiple objectives, especially when their units or orders of magnitude were distinct (Zhu et al., 2014; Ji, 2017).

To address the prescribed shortcomings of multi-objective programming methods, fractional programming (FP) has attracted more and more attentions in recent years. FP optimized the ratio of two objectives with their original magnitudes, such as output/input, benefit/volume or benefit/time (Mehra et al., 2007; Zhou et al., 2015). It

* Corresponding author at: College of Water Resources and Civil Engineering, China Agricultural University, 17 Qinghua East Road, Beijing 100083, China.
E-mail address: qian_tan@cau.edu.cn (Q. Tan).

https://doi.org/10.1016/j.jhydrol.2018.07.080
Received 21 May 2018; Received in revised form 21 July 2018; Accepted 31 July 2018
Available online 01 August 2018
0022-1694/

could thus provide an unprejudiced method for measuring system efficiency and effectively optimize marginal system benefits (Cui et al., 2015). FP could better fit real-world problems when the objective function is the quotient of physical and economic quantities (Emam, 2013). Charnes and Cooper (1962) first developed a classic algorithm for linear fractional programming. Since then, a great deal of work has been carried out. The latest progress was reported in a review paper by Stancu-Minasian (2017). In the past five decades, FP has been widely used for increasing system efficiency in many fields such as financial and production planning (Mehra et al., 2007). A few reported studies applied the FP method to address agricultural water management problems. For instance, Amini Fasakhodi et al. (2010) assessed water resources sustainability and optimized cropping patterns through a multi-objective fractional programming model. Mani et al. (2016) optimized the conjunctive use of groundwater and surface water resources through a mixed integer linear fractional programming method. Zhao et al. (2017) proposed a linear fractional programming model to coordinate multiple factors and promote agricultural water productivity. Although FP was proven effective for measuring the efficiency of agricultural water resources system, uncertainties that were prevalent in all components of agricultural water management systems could not be realistically reflected in the previous FP studies.

Agricultural water management systems involve plenty of uncertain factors and parameters that can hardly be expressed as deterministic values, such as uncertain irrigation water requirements, economic revenues and crop yields (Gui et al., 2017; Li et al., 2013, 2015; Liu et al., 2014; Zeng et al., 2014; Dong et al., 2015). Uncertain parameters should thus be tackled in the planning models. There were a few FP-based studies dealing with uncertainty in agricultural water management. For example, Guo et al. (2014) tackled multiple uncertainties in agricultural water management problems that were expressed as probability distributions, fuzzy sets, and their combinations through formulating a fuzzy chance-constrained linear fractional programming model. Cui et al. (2015) addressed stochastic uncertainty in irrigation water management systems through the development of a two-stage stochastic fractional programming model. Li et al. (2016) handled interval uncertainties and improved irrigation water productivity through the formulation of an interval fractional programming model. Zhang and Guo (2017) dealt with fuzziness existing in irrigation water allocation problems based on a generalized fuzzy credibility-constrained linear fractional programming method. Fu et al. (2017) handled stochastic water availability and optimized agricultural water-use structure through integrating chance-constrained programming, conditional value-at-risk, and fractional programming. Most of them dealt with uncertainties expressed as fuzzy sets and/or stochastic variables with unknown possibility and/or probabilistic distributions (Stancu-Minasian, 2017; Tan et al., 2015). Nevertheless, these fractional programming studies could not deal with uncertain parameters without detailed distributions nor quantify system-violation risks arising from such uncertainties. To provide reliable and risk-explicit solutions under deep uncertainty without distribution information, robust optimization could be a promising way. Robust optimization focused on solving the worst case with the most unfavorable uncertainty (Gabrel et al., 2014). In its early development stage, the algorithms were mostly ultra-conservative and difficult to solve (Naderi and Pishvaee, 2017). Bertsimas and Sim (2003) proposed a robust optimization method that allowed controlling the degree of conservatism and was relatively easy for computation. Robust optimization has been used in the planning of asset inventory, finance, and energy systems (Gabrel et al., 2014; Dong et al., 2013). For instance, Palma and Nelson (2010) applied the robust optimization method to support decision-making on forest harvest scheduling. Tan et al. (2010) developed a radial-interval linear programming to tackle highly uncertain information in waste management problems. Parisio et al. (2012) applied robust optimization methods to address an energy hub operation problem. However, robust optimization methods were hardly used to address agricultural water management problems under uncertainty, especially when multiple objectives were involved.

Therefore, this paper aims at developing a robust fractional programming (RFP) method that not only deals with ratio multi-objective problems but also enhances the robustness of optimal plans for supporting agricultural water management under complex uncertainties. The applicability of the developed RFP method will be demonstrated through a case study of agricultural water management problems in arid and semi-arid region of China. The obtained modeling results could provide decision makers with optimal adjustment plans for crop planting structures, and inform them with associated water-shortage risks. This paper is organized as follows: the development of the RFP method is explained in Section 2; the developed RFP method is applied to a real-world case in north-west China in Section 3; in Section 4, the results are interpreted and compared; validation and comparisons are discussed in Section 5; and Section 6 provides concluding remarks of this research.

2. Development of robust fractional programming method

In many real-word problems, there usually exist multiple hard-to-coordinate objectives, such as maximized economic income and minimized resources input. One of the ultimate solutions for coordinating these conflicting objectives is to strengthen system efficiency, which can essentially be considered as a ratio optimization problem. At the same time, uncertainty that is ubiquitous in the processes, components and factors of agricultural water management systems cannot be ignored. In many cases, precise fluctuation information of uncertain parameters is difficult to obtain. To overcome the inabilities of existing programming methods in simultaneously tackling ratio optimization problems and complex uncertainties, a RFP method is proposed in this paper. In RFP, the objective function is expressed as the outputs divided by the inputs. Moreover, uncertain parameters without detailed distribution information are coped with in the constraints.

A general RFP model can be formulated as follows:

$$\max f = \frac{\sum_{j=1}^{n} c_j x_j + \alpha}{\sum_{j=1}^{n} d_j x_j + \varepsilon} \tag{1a}$$

subject to:

$$\sum_{j=1}^{n} \overleftrightarrow{a}_{ij} x_j \leq \overleftrightarrow{b}_i \quad i \in M,\ i \neq k \tag{1b}$$

$$\sum_{j=1}^{n} a_{kj} x_j \leq b_k \quad k \in M,\ k \neq i \tag{1c}$$

$$x_j \geq 0 \quad j = 1, 2, \ldots, n \tag{1d}$$

where a_{kj}, b_k, c_j, d_j, $x_j \in R$; $\overleftrightarrow{a}_{ij}$, $\overleftrightarrow{b}_i \in \overleftrightarrow{R}$; α and ε are scalar constants; R is the set of real numbers; and $\overleftrightarrow{R}$ donates the set of uncertain variables with known fluctuation ranges but unknown symmetric distributions. The values of such uncertain coefficient $\overleftrightarrow{a}_{ij}$ fall within a fluctuation range $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$, where $\bar{a}_{ij}$ represents the deterministic nominal value of $\overleftrightarrow{a}_{ij}$ and $\hat{a}_{ij}$ is the fluctuation radius around the nominal value that determines the size of fluctuation range.

Assume that the objective function (1a) is constantly differentiable and the solution set is nonempty and bounded (Zhu et al., 2014). The objective function can be transformed into a linear expression and solved by introducing a new variable r ($r \in R$), under the condition that the denominator is positive and constant in sign (Zhu et al., 2014; Charnes and Cooper, 1962; Mani et al., 2016). Model (1) can thus be transformed to its linear form as follows:

$$\max g(y_1, y_2, \ldots, y_n, r) = \sum_{j=1}^{n} c_j y_j + \alpha r \tag{2a}$$