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Review papers

Cellular Automata and Finite Volume solvers converge for 2D shallow flow modelling for hydrological modelling

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ABSTRACT

Surface flows of hydrological interest, including overland flow, runoff, river and channel flow and flooding have received significant attention from modellers in the past 30 years. A growing effort to address these complex environmental problems is in place in the scientific community. Researchers have studied and favoured a plethora of techniques to approach this issue, ranging from very simple empirically-based mathematical models, to physically-based, deductive and very formal numerical integration of systems of partial-differential equations. In this work, we review two families of methods: cell-based simulators – later called Cellular Automata – and Finite Volume solvers for the Zero-Inertia equation, which we show to converge into a single methodology given appropriate choices. Furthermore, this convergence, mathematically shown in this work, can also be identified by critically reviewing the existing literature, which leads to the conclusion that two methods originating from different reasoning and fundamental philosophy, fundamentally converge into the same method. Moreover, acknowledging such convergence allows for some generalisation of properties of numerical schemes such as error behaviour and stability, which, importantly, is the same for the converging methodology, a fact with practical implications. Both the review of existing literature and reasoning in this work attempts to aid in the effort of synchronising and cross-fertilizing efforts to improve the understanding and the outlook of Zero-Inertia solvers for surface flows, as well as to help in clarifying the possible confusion and parallel developments that may arise from the use of different terminology originating from historical reasons. Moreover, synchronising and unifying this knowledge-base can help clarify model capabilities, applicability and modelling issues for hydrological modellers, specially for those not deeply familiar with the mathematical and numerical details.

1. Introduction

The simulation of spatially-distributed and time-dependant surface flow processes – with interests on flood modelling, runoff modelling and geomorphology – has been approached with different levels of complexity. One of the simplest and general approaches are cell-based methods, often (but not always) termed the Cellular Automata (CA) approach, originally proposed by Von Neumann (1966) in the context of computationally mimicking biological behaviours. In this approach, an individual automaton – a discrete entity with properties – communicates with its neighboring automata through some prescribed rules of interaction (Fonstad, 2006) – which may be argued to be fluxes. Clearly, this requires to define what is meant by “neighborhood” and what are such rules, which in turn, obviously depends on the intended application. Within the plethora of applications for CA models, only those on runoff and surface flow phenomena are of interest to this work. One of the earliest works in this context is that of Murray and Paola (1994) who attempted to model braided rivers using CA. In their approach, CA was used to discretise space and interaction rules were implemented for both water and sediment flows. Such rules depended basically on bed slope, and were rather convenient conceptual formulations. Other authors have chosen different intercell fluxes, depending on their interest. For example, Cai et al. (2014) chose a broad-crested weir rating curve to describe the intercell flux. Ghimire et al. (2013) proposed fluxes based on a cascade volume transfer strategy among pre-determined cell sets, including a relaxation parameter to aggressively damp numerical oscillations. On the other side, models spawning from simplified shallow-water dynamics have also been present. Solving the full shallow water equations (SWE) can be challenging and computationally costly, in particular for large domains typical of hydrological problems (García-Navarro, 2016). Although this issue is currently being addressed through the development of advanced numerical strategies and the use parallel and GPU (Graphical Processing Unit) computing (Kesserwani et al., 2016), alternative

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Table 1
CA and FV models.

Reference	Termed	Discretisation		Flux	N	Application	Comment
		Space	Time				
Murray and Paola (1994)	CA	CA	E	Ad-hoc routing	DS	Braided river morphology	Routs to three downstream cells
Julien et al. (1995)	RB	FV	E	ZI	VN	Hydrological modelling	Known as CASC2D
Lal (1998)	FD	FD	E/I	ZI	–	Overland flow	Unstructured meshes
Bates and Roo (2000)	RBSC	FV/CA	E	ZI	VN	Flood modeling	Includes 1D kinematic wave solver
D'Ambrosio et al. (2001)	CA	CA	E	Gradient minimization	VN	Erosion	–
Thomas and Nicholas (2002)	Cellular routing	CA	E	Ad-hoc routing	DS	Braided river morphology	Routs to five downstream cells
D'Ambrosio et al. (2003)	CA	CA	E	Gradient minimization + kinetic energy	–	Debris flow	Hexagonal cells
Panday et al. (2004)	FD-ZI	FD	I	ZI	VN	Coupled surface-subsurface flow	Manning, Darcy and Chezy friction laws
Hunter et al. (2005)	RBSC	FD	E	ZI	VN	Flood modelling	Stability issues on complex topography
Kollet and Maxwell (2006)	FV	FV	I	Kinematic wave	VN	Coupled surface-subsurface flow	Rainfall-runoff assessment
Parsons and Fonstad (2007)	CA	CA	E	Manning	VN	Rainfall-runoff	–
Rinaldi et al. (2007)	CA	CA	E	Downslope routing	M	Flood modelling	–
Wiel et al. (2007)	CA	CA	SE	Water surface slope + Manning	VN	Alluvial Geomorphology	Flow-sweep algorithm
Weill et al. (2009)	FE	FE	I	ZI	–	Coupled surface-subsurface flow	ZI-Richards similarity (Generalized Richards)
Cea et al. (2010)	FV-DW	FV	E	ZI	VN	Urban rainfall-runoff	Square and triangular cells
Bates et al. (2010)	Inertial	FD	E	ZI + inertia	VN	Flood modelling	Keeps an inertial term
Aricò et al. (2011)	FV-DW	FV	E	ZI	–	Flood modelling	Fractional time-stepping
Dottori and Todini (2011)	CA	FV	E	ZI	VN	Flood modelling	Local time stepping
Wang et al. (2011)	ZI	FV	E	ZI	VN	Flood modelling	Low CFL may be enough for stability
Lopez-Barrera et al. (2012)	FV	FV	E	ZI	VN	Hydrological modelling	Hyperbolic-like approach
Baartman et al. (2012)	CA	CA	E	Gradient based	VN	Land evolution	–
Ghimire et al. (2013)	CA	CA	E	Ranked-cells outflow scheme	VN	Urban pluvial flood modelling	Velocity limited by Manning
Cai et al. (2014)	CA	CA	E	Weir rating curve	M	Flood modelling	–
Leandro et al. (2014)	FV	FV	E	ZI	VN	Flood modelling	Stability study and parallelization
Mendicino et al. (2015)	Macroscopic CA	CA	E	ZI	VN	Ecohydrological simulation	Stability discussion
Shao et al. (2015)	CA	CA	E	Travel time + Manning	M	Rainfall-runoff	–
Liu et al. (2015)	CA	CA	E	Gradient minimization + Manning	VN	Urban floods	–
Fernández-Pato and García-Navarro (2016)	FV-ZI	FV	E/I	ZI	–	Rainfall-runoff	Triangular cells
Costabile et al. (2017)	FV-DW	FV	E	ZI	–	Urban flood modelling	Triangular cells
Jahanbazi et al. (2017)	OFS-CA	FV	E	ZI	VN	Flood modelling and runoff	Novel corrections for stability and efficiency

CA: Cellular Automata, FV: Finite Volumes, FD: Finite Differences, FE: Finite Elements, RB: Raster-based, RBSC: Raster-based storage cell, ZI: Zero-Inertia, DW: Diffusive-wave, N: Neighborhood; VN: von Neumann, M: Moore, DS: Downslope, E: Explicit, I: Implicit, SE: Semi explicit.

approaches, which are mathematically, numerically and computationally simpler have also been historically adopted to make simulation of these types of problems feasible and accessible. Various studies have explored the Zero-Inertia (ZI) – also often inaccurately termed Diffusive Wave (Yen and Tsai, 2001) – model, with different numerical strategies (Costabile et al., 2017; Dottori and Todini, 2011; Fernández-Pato et al., 2016; Julien et al., 1995; Panday et al., 2004, e.g.).

The growing and very recent literature on both cellular-automata (CA) and finite-volumes (FV) based solvers clearly indicates that this remains an active field, and that an effort is required so that several communities and methods may effectively converge. Most importantly, the growing use of these models for sophisticated, spatially distributed, large scale hydrological simulation prompts the need to robustly identify key advantages and disadvantages of the underlying numerical approaches, and requires for modellers to be deeply aware of the applicability and assumptions of the models available to them and a working understanding of the underlying numerics of their computational tools. This needs motivate this review. In particular, we set out to draw attention and clearly show how some so-called Cellular-Automata solvers are exactly the same as the explicit Finite Volume solvers of the ZI equation. This is the main contribution of this work, which has the significant implication that a vast knowledge base can be brought together thus aiding hydrological modellers to better understand the available computational tools. In particular, it is noteworthy that stability and error properties of the two solvers are the same and well known, and that they differ from the less-studied stability properties of other – perhaps less formal – cell-based routing models. In order to advocate that effort, it is our goal to review and summarize the contributions from both communities to what is in fact, the same numerical approach to the same mathematical approximation of the shallow water equations. To do so, we derive both a CA simulator from fundamental discrete principles in Section 2, we derive the mathematical and numerical expressions for a FV-ZI solver in Section 3 and we finally show and discuss the equivalence between methods, review and classify a plethora of existing models reported as CA and FV in the literature, and compare and contrast them in Section 4. 5 briefly summarises key insights and outlook.

2. The Cellular Automata approach for surface flows

A general form for the CA state evolution rule (Cai et al., 2014) for a state variable $\mathscr{S}$ is

$$\mathscr{S}_i^{n+1} = f(\mathscr{S}_i^n, \mathscr{S}_j^n) \quad (1)$$