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Comparative study of 1D entropy-based and conventional deterministic velocity distribution equations for open channel flows

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ABSTRACT

Velocity distributions for open channel flows have been investigated using deterministic and probabilistic approaches. It is well known that the vertical velocity profiles in wide open channels (i.e. aspect ratio width/depth > 5) can be approximated by logarithmic velocity laws and power laws. Recently the entropy concept in the forms of Shannon entropy and Tsallis entropy has been employed to estimate velocity distributions in open channels with different aspect ratios. The accuracy of conventional velocity equations is highly dependent on their parameters that can only be estimated by empirical or semi-empirical analytical relations which requires either a good knowledge of velocity field and/or physical properties of the channel, such as topographic conditions, sedimentation conditions and boundary roughness. In contrast, the entropy based velocity distributions derived based on the least-biased probability density function (PDF) by treating time-averaged velocities as random variables are resilient regardless of the flow and channel conditions. However, a comparison of the velocity profiles computed using deterministic approaches and probabilistic approaches has not been rigorously conducted. Furthermore, the accuracy and reliability of associated velocity distribution equations have not been tested thoroughly using data sets collected using advanced techniques. This paper presents a comprehensive and comparative study to analyze the distinctions and linkages between four commonly used velocity laws and two entropy-based velocity distributions theoretically and quantitatively using selective laboratory and field measurements available in the literature, considering typical sedimentation and channel hydraulic conditions. Amongst all, Tsallis entropy based velocity distribution developed from a generalized form of informational entropy exhibits universal validity to sediment-laden flows in wide alluvial open channels, and is found to be superior to others to predict velocity profiles in large waterways with unmanageable rough beds.

1. Introduction

There is a shortage of reliable velocity data in waterway system during storm events, in which the discharge varies rapidly and velocity sampling is extremely difficult. Recently, high-tech based measuring techniques, such as Particle Image Velocimetry (PIV) (Adrian, 1997; Hyun et al., 2003), Acoustic Doppler Velocimeters (ADV) (Holmes and Garcia, 2008), Acoustic Doppler Current Profilers (ADCP) (Gonzalez et al., 1996), and previously developed laser Doppler velocimetry (LDV), also known as laser doppler anemometry (LDA) (Nezu and Rodi, 1986), have been applied, however, their implementation is still subject to uncertainties associated with weather and measurement techniques and involve considerable cost. Furthermore, velocities near solid boundaries and free surface cannot be measured with ADCP's due to the interference in the acoustic signals caused by boundary reflectance, and are extrapolated based on the power law distribution currently incorporated in ADCP post-processing software. Thus, for these measurements to be efficient, a reliable estimation of velocity distribution for the entire flow depth that has wide applicability regardless of channel properties and flow conditions is preferable to sufficiently reduce the number of velocity observations.

Herein, the velocity distribution refers to the vertical distribution of time-averaged velocity of primary flow in a transverse cross-section, eliminating the turbulent oscillation component. Nezu and Rodi (1986) found experimentally that the open channel flow velocity distribution is highly dependent on the presence of secondary currents due to the side wall effects. They further divided open channels into two categories as "narrow" or "wide" channels according to the aspect ratio $A_r \equiv B/D$. For narrow channels with $A_r \leqslant 5$, the maximum velocity happens beneath the water surface as a result of the momentum exchange between the primary flow and the secondary circulation (also known as "velocity dip "); for wide channels with $A_r > 5$, the side wall effects are,

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however, reduced and become negligible in the center zone of the width $B-5D$. Thus, for sufficiently wide channels with $A_r \gg 5$, the velocity distribution in the large portion of the cross-section can be considered to follow a 1D type of velocity distribution, in which the velocity profile along each vertical is a monotonic increasing function of the vertical distance to the bed. The current analysis is confined to wide channels where 1D assumptions are valid.

In fluid mechanics, open-channel flows are part of the general class of bounded shear flows with free surface, where velocity profiles are obtained by solving Navier-Stokes equations with given boundary conditions usually under steady uniform flow assumptions. Logarithmic and power laws are two well-known forms of such solution family. von Karman (1921, 1930) and Prandtl (1925) first investigated flow through pipes theoretically by integrating the experimental studies of Nikuradse (1933), and derived a set of rational velocity distributions and hydraulic-resistance relations for turbulent flows over flat plates and in circular pipes. These have been extended to open channel flow to include free surface effects (Rouse, 1959; Sarma et al., 1983). Furthermore, extensive research has been done to extend the logarithmic-type velocity or velocity-defect law to sediment-laden flow by treating the von Karman constant and the "law of the wake" as functions of flow, sediment-load characteristics (Vanoni, 1946; Coles, 1956; Einstein and Chien, 1955; Chien, 1956; Yanlin and Finlayson, 1972; Schlichting, 1979; Coleman, 1981; Parker and Coleman, 1986; Wang, Apr 1981; Sill, 1982; Hoffman and Mohammadi, 1991; Chiu and Murray, 1992).

Hinze (1959) showed that the logarithmic velocity distribution can be approximated by the power-law velocity distribution in most of the boundary layer cross section, specially in the overlapping region of the inner law (i.e. the law of the wall) and the outer law (i.e., the velocity defect law) (Chen, 1991). Wooding et al. (1973) then theoretically evaluated the differences between the logarithmic velocity distribution and the power velocity distribution and found that the power law with a small exponent cannot be distinguished experimentally from the logarithmic law. It was argued that the power-law velocity profile was preferable to the logarithmic velocity distribution (Landweber, 1957; Rouse, 1959) for three reasons: (1) the logarithmic velocity distribution should be considered as an asymptotic law valid for very large Reynolds numbers (Schlichting, 1968), whereas, the power law is less restrictive to the flow regime and is also applicable to smaller Reynolds numbers (Chen, 1991); (2) the power law appears to be better able to incorporate the effects of sediment on velocity profiles without troublesome singularities near the bed and/or derivative discontinuities at axes and planes of symmetry (Karim and Kennedy, 1987; Chen, 1991); (3) Hinze (1975) and Schlichting (1979) showed that the power law conforms better than the logarithmic relation to the pipe-flow data of Nikuradse (1933) and Laufer (1953).

However, due to the inherent system randomness and incomplete input information, there are always uncertainties associated with theoretical predictions or experiment-based calculations of variables or parameter estimation involved in the above conventional deterministic approaches. Thus, the time-averaged streamwise velocities should rather be statistically regarded as a random variable. The informational entropy theory facilitates the application of probabilistic approaches to hydraulics and hydrology as well as environmental engineering by accounting for the associated uncertainties (Singh, 2013; Singh, 2014; Singh, 2015). The Principle of Maximum Entropy (**POME**) (Jaynes, 1957; Jaynes, 1957) states that any system in equilibrium state tends to maximize its entropy under universal constraints which is equivalent to the theory of minimum energy dissipation (Yang, 1976). Chiu (1987) first applied the probabilistic concept via Shannon entropy to describe the vertical distribution of mean velocity, shear stress, and suspended sediment concentration in open-channel flows using Shannon Entropy subject to POME. Thus, the derived velocity distribution is claimed to be least-biased towards the unknowns and most biased towards the constraints (Marini et al., 2011). Tsallis entropy, a generalized form of informational entropy, was also utilized to describe the streamwise velocity in wide-open channels (Singh and Luo, 2011; Cui and Singh, 2014; Singh, 2016).

Besides the difference in mathematical formulations of the numerous developed velocity distribution equations, no comprehensive and rigorous analysis has been conducted to compare the goodness-of-fit of the aforementioned methods using high resolution data measured with advanced tools and techniques. Thus, this paper presents a comparative analysis on the widely accepted forms of 1D logarithm, power law velocity distribution and two entropy-based velocity distributions both theoretically and quantitatively, with the following specific objectives: (1) to compare preselected feasible ways that well balance information parsimony and overall accuracy to estimate key parameters involved in logarithmic and power-law velocity distributions; (2) to identify the differences and linkages between the formulations of different methods and (3) to compare the accuracy and applicability of the velocity-distribution laws considered under various flow and channel conditions. Overall, this paper demonstrates the resilience of entropy-based velocity distributions in variety of applications and provides auxiliary alternatives for streamflow measurement and numerical validation purposes.

2. Conventional deterministic 1D velocity distributions

2.1. Logarithmic velocity Distribution

The 1D logarithmic velocity law was developed by assuming a steady, uniform, in a wide open-channel flow having a mean width B and a mean depth D (i.e. aspect ratio defined as $A_r = B/D \geqslant 5$) with a mean flow velocity U as shown in Fig. 1. The flow velocity distribution under consideration was well established, both experimentally and from dimensional analysis (Schlichting, 1979; Nezu and Rodi, 1986) as:

$$u = \frac{u_*}{\kappa}\ln\frac{y}{y_0} \tag{1}$$

Eq. (1) is considered as the "law of the wall " since it was considered to strictly apply inherently in a relatively thin layer ($y/D < 0.2$) near the bed (Coleman and Alonso, 1983; Nezu and Nakagawa, 1993); whereas, it is commonly used as a reasonable estimation of the entire depth for most of the flow in many streams and rivers with a correction for wake effects(Coles, 1956; Coleman, 1986) which can be ignored for single-phase and uniform flows.

Schlichting (1979) derived the outer form of the law of the wall by manipulating Eq. (1) based on the assumption that the maximum flow velocity u_{max} takes place at the water surface where $y = D$ (i.e. $u_D = u_{max}$). This alternative formulation of logarithmic velocity law is also known as velocity-defect law, whereas, it does not require as much knowledge of the bed roughness as the "law of the wall" does.

$$\frac{u_{\max}-u}{u_*} = -\frac{1}{\kappa}\ln\left(\frac{y}{D}\right) \tag{2}$$

Parameter estimation: Key parameters in Eq. (1) are shear velocity (u_*), von-Karman constant (κ), and bed roughness length (y_0) which equates to a very small (almost unmeasurable) value of depth above the bed where the flow velocity goes to zero. Based on the experimental results by Nikuradse (1933),Christoffersen and Jonsson (1985) developed a generalized formula to estimate y_0 as a function of effective roughness height k_s for flows from hydraulic transitional to fully hydraulic rough as:

$$y_0 = \frac{k_s}{30}\left[1-\exp\left(\frac{-u_*k_s}{27\nu}\right)\right] + \frac{\nu}{9u_*} \tag{3}$$

where ν is the kinematic viscosity of water. The roughness k_s reflects both surface roughness and form roughness of the channel bed (Rouse, 1959), hence, its complete evaluation is complicated which further limit the accuracy of Eq (3). Singh (2011) proposed an even more simple algebraic approach by substituting $y = D$ in Eq. (1), given the