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Review papers

Review of analytical models to stream depletion induced by pumping: Guide to model selection

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ABSTRACT

Stream depletion due to groundwater extraction by wells may cause impact on aquatic ecosystem in streams, conflict over water rights, and contamination of water from irrigation wells near polluted streams. A variety of studies have been devoted to addressing the issue of stream depletion, but a fundamental framework for analytical modeling developed from aquifer viewpoint has not yet been found. This review shows key differences in existing models regarding the stream depletion problem and provides some guidelines for choosing a proper analytical model in solving the problem of concern. We introduce commonly used models composed of flow equations, boundary conditions, well representations and stream treatments for confined, unconfined, and leaky aquifers. They are briefly evaluated and classified according to six categories of aquifer type, flow dimension, aquifer domain, stream representation, stream channel geometry, and well type. Finally, we recommend promising analytical approaches that can solve stream depletion problem in reality with aquifer heterogeneity and irregular geometry of stream channel. Several unsolved stream depletion problems are also recommended.

1. Introduction

A considerable amount of stream water may flow toward a nearby well due to pumping in an aquifer, which is referred to as stream depletion. Stream depletion is an important issue that attracts the attention of hydrologists because it may have impacts on aquatic ecosystems (e.g., Foglia et al., 2013). Streambed may become completely dry because of stream depletion (e.g., Hunt, 2014). Contaminants in a polluted stream may enter the adjacent aquifer and arrive at irrigation wells (e.g., Chen 2001). Moreover, groundwater abstraction from a well near two streams may involve conflict over water distribution regulated by water rights (e.g., Sun and Zhan, 2007).

The stream depletion rate (SDR), defined as a dimensionless ratio of the volumetric rate of water abstraction from a stream to a pumping rate, is an index to quantify stream depletion. The SDR ranges from zero to unity if all interconnected surface-water features are considered. The SDR may be less than unity if derived from a particular stream or boundary segment. Reeves et al. (2009) developed Michigan Water-Withdrawal Screening Tool based on the analytical solution of Hunt (1999) to assess resource impacts due to water withdrawal. Recently, Hunt (2014) reviewed analytical solutions of SDR and provided a useful tool for analyzing groundwater resources, Function.xls, which can be downloaded from his website.

A large number of articles have addressed the problem of stream depletion, but a general framework illustrating the applicability and limitations in various analytical models from aquifer viewpoint has not been presented. This review illustrates key differences in existing models and serves as a guide to select an appropriate analytical model for engineering applications. The target audiences are those who apply analytical approaches to solve real-world problems of stream depletion. At the beginning, we provide a mathematical framework composed of flow equations along with associated boundary conditions (BCs) and source/sink terms for pumped confined, unconfined and leaky aquifers near a stream. Then, we give some comments on analytical models regarding stream depletion problem. Finally, we recommend some unsolved problems that may be of practical use in the future.

2. Framework of problem

2.1. Flow equations

The notations used in the text are summarized in Table 1. Fig. 1 illustrates typical stream-aquifer systems in panels for (a) a confined aquifer, (b) an unconfined aquifer, (c) a leaky aquifer with an overlying source bed, (d) a multilayered system and (e) a two-layered system with water table in the aquitard. In each of the panels, the stream partially

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Table 1
Notations, acronyms and their descriptions.

Notation (unit)	Description
a (L)	Distance between stream and well
BC	Boundary condition
B' (L)	Aquitard thickness
B_i' (L)	Thickness of aquitard beneath i-th layer of aquifer system
B'' (L)	Streambed thickness
D (L)	Aquifer thickness
D_i (L)	Thickness of i-th layer of aquifer system
d (L)	Stream penetration depth from the top of aquifer
FPS	Fully penetrating stream
FPW	Fully penetrating well
h (L)	Hydraulic head in aquifer
h_a (L)	Hydraulic head in source bed
h_i (L)	Hydraulic head in i-th layer of aquifer system
h_0 (L)	Total head of stream stage
h' (L)	Hydraulic head in aquitard
K_h (L/T)	Isotropic aquifer hydraulic conductivity in horizontal direction
K_i' (L/T)	Hydraulic conductivity of aquitard beneath i-th layer of aquifer system
K_n (L/T)	Aquifer hydraulic conductivity in n direction normal to stream-aquifer interface
K_x, K_y, K_z (L/T)	Aquifer hydraulic conductivities in x-, y- and z-directions, respectively
K' (L/T)	Aquitard hydraulic conductivity
K'' (L/T)	Streambed hydraulic conductivity
N	The number of layers of aquifer system
n	Direction normal to stream-aquifer interface
P (T)	Pumping period
PPS	Partially penetrating stream
PPW	Partially penetrating well
Q (L^3/T)	Pumping rate
RCW	Radial collector well
RHS	Right-hand side
r (L)	Radial distance from the well center
r_c, r_w (L)	Inner and outer well radius, respectively
S	Aquifer storage coefficient
SDR	Stream depletion rate
S_i	Storage coefficient of i-th layer of aquifer system
S_s (L^{-1})	Aquifer specific storage
S_y	Aquifer specific yield
S_y'	Aquitard specific yield
T (L^2/T)	Aquifer transmissivity
T_i (L^2/T)	Transmissivity of i-th layer of aquifer system
t (T)	Time since pumping
$\bar{t}$	Dimensionless time defined as $T_1t/(S_1a^2)$ for Fig. 2 and $Tt/(Sa^2)$ for Fig. 3
U(·)	Unit step function
w (L)	Stream channel width
x, y, z (L)	The Cartesian coordinate
x_0, y_0, z_0 (L)	Point sink location
z_l, z_u (L)	Lower and upper elevations of well screen, respectively
$\delta(\cdot)$	Dirac delta function
κ'', $\kappa_{2,1}$	$K''a/(K_hB'')$, T_2/T_1
$\sigma_{2,1}$	S_2/S_1
λs	$K'a^2/(TB')$
Ω_s	Domain of stream-aquifer interface or streambed
Subscript i	Integers, i.e., 1, 2, 3,…

Note: L and T in brackets represent length and time units, respectively.

penetrates the aquifer with low permeability streambed in between. The stream-aquifer systems overlie an impervious stratum. The penetration degree defined as the ratio of stream penetration depth to aquifer thickness (i.e., d/D) is equal to unity for a fully penetrating stream (FPS) and less than unity for a partially penetrating stream (PPS). The following outlines flow equations for the stream-aquifer systems.

The flow equation describing three-dimensional (3D) transient hydraulic head $h(x, y, z, t)$ in a homogeneous aquifer can be expressed as (e.g., Bear, 1979)

$$K_x\frac{\partial^2 h}{\partial x^2}+K_y\frac{\partial^2 h}{\partial y^2}+K_z\frac{\partial^2 h}{\partial z^2}=S_s\frac{\partial h}{\partial t} \tag{1}$$

Aquifer hydraulic conductivities are typically isotropic in the horizontal directions (i.e., $K_x = K_y = K_h$), and the value of K_z/K_h ranges between 0.1 and 0.5 for alluvium and is possibly as low as 0.01 for clay layers (Todd and Mays, 2005).

The equation for vertically integrated two-dimensional (2D) transient flow is written as (e.g., Bear, 1979)

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S}{T}\frac{\partial h}{\partial t} \tag{2}$$

Eq. (2) is applicable only to a confined aquifer connected to a FPS. On the other hand, Eq. (2) can be used to simulate 2D flow in unconfined aquifers when applying the Dupuit assumption, replacing S by specific yield S_y, and assuming $T = K_hD$ with D being averaged aquifer thickness. Huang et al. (2016) reported that an estimated SDR based on 2D unconfined flow is accurate when $K_za^2/(K_hD^2) \geqslant 30$ with a being distance between a well and a stream.

The quasi 3D flow in a multilayered aquifer system illustrated in panel (d) of Fig. 1 assumes horizontal 2D flow in the aquifers and vertical flow in the aquitards. The equation describing 2D flow in each aquifer can be written as (e.g., Hunt, 2009)

$$\frac{\partial^2 h_i}{\partial x^2}+\frac{\partial^2 h_i}{\partial y^2}=\frac{S_i}{T_i}\frac{\partial h_i}{\partial t}+\frac{K_i'}{T_iB_i'}(h_i-h_{i+1})+\frac{K_{i-1}'}{T_iB_{i-1}'}(h_i-h_{i-1})\text{ for } i\in(1,2,\ldots N) \tag{3}$$

where the subscript i represents the i-th layer from the top. The second term on the right-hand side (RHS) of Eq. (3) quantifying leakage through the underlying aquitard should be removed when $i = N$.Similarly, the third RHS term describing leakage through the overlying aquitard should be deleted if $i = 1$. The water table decline in the top aquitard shown in panel (e) of Fig. 1 can be expressed as (Hunt, 2003)

$$S_y'\frac{\partial h'}{\partial t}+\frac{K'}{B'}(h'-h_1)=0 \tag{4}$$

that describes vertical flow in the aquitard and couples Eq. (3) where $i = 1$ and the third RHS term is replaced by $K'(h_1-h')/(T_1B')$.

2.2. Boundary conditions

2.2.1. Free surface equation

A linearized free surface equation describing water table movement illustrated in panel (b) of Fig. 1 can be written as (Neuman, 1972)

$$S_y\frac{\partial h}{\partial t}=-K_z\frac{\partial h}{\partial z}\text{ at } z=D \tag{5}$$

where $h > 0$ for a rise of water table and $h < 0$ for a decline. When $S_y = 0$, Eq. (5) reduces to $\partial h/\partial z = 0$ which can be regarded as a no-flow BC at impervious strata. Note that Eq. (5) is specified at the fixed elevation of $z = D$ and applicable when $|h|/D \leqslant 0.1$ for small movement of water table relative to the initial aquifer thickness (Huang et al., 2016).

2.2.2. Leakage across aquitard

Consider a two-layered aquifer system with an aquitard in between as shown in panel (c) of Fig. 1. The upper aquifer is assumed to be a source with constant hydraulic head h_a. The flux through the aquitard satisfies (e.g., Zhan and Park, 2003):

$$K_z\frac{\partial h}{\partial z}=\pm\frac{K'}{B'}(h-h_a)\text{ at } z=D \tag{6}$$

where h is the hydraulic head in the lower aquifer. The sign ± is negative for $\partial h/\partial z > 0$ and positive for $\partial h/\partial z < 0$. Eq. (6) can be regarded as a top BC for Eq. (1) describing 3D flow in the lower aquifer. In the absence of vertical flow, the source term $K'(h-h_a)/(TB')$ can be inserted into the RHS of Eq. (2) to account for the effect of leakage on 2D flow in