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Influence of Boussinesq coefficient on depth-averaged modelling of rapid flows

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ABSTRACT

The traditional Alternating Direction Implicit (ADI) scheme has been proven to be incapable of modelling trans-critical flows. Its inherent lack of shock-capturing capability often results in spurious oscillations and computational instabilities. However, the ADI scheme is still widely adopted in flood modelling software, and various special treatments have been designed to stabilise the computation. Modification of the Boussinesq coefficient to adjust the amount of fluid inertia is a numerical treatment that allows the ADI scheme to be applicable to rapid flows. This study comprehensively examines the impact of this numerical treatment over a range of flow conditions. A shock-capturing TVD-MacCormack model is used to provide reference results. For unsteady flows over a frictionless bed, such as idealised dam-break floods, the results suggest that an increase in the value of the Boussinesq coefficient reduces the amplitude of the spurious oscillations. The opposite is observed for steady rapid flows over a frictional bed. Finally, a two-dimensional urban flooding phenomenon is presented, involving unsteady flow over a frictional bed. The results show that increasing the value of the Boussinesq coefficient can significantly reduce the numerical oscillations and reduce the predicted area of inundation. In order to stabilise the ADI computations, the Boussinesq coefficient could be judiciously raised or lowered depending on whether the rapid flow is steady or unsteady and whether the bed is frictional or frictionless. An increase in the Boussinesq coefficient generally leads to overprediction of the propagating speed of the flood wave over a frictionless bed, but the opposite is true when bed friction is significant.

1. Introduction

The increasing risk of flooding, especially flash flooding, has become a prevalent and pressing issue worldwide. Flash floods involve high flow velocities, which can be very destructive and pose a serious risk to both infrastructure and human life. Therefore, it is extremely important that such flooding events can be predicted accurately.

The horizontal scale of a flood is normally much larger than its depth. This type of free surface flow problem can be mathematically described by the Shallow Water Equations (SWEs). These equations can be derived from the depth integration of the three-dimensional incompressible Navier–Stokes equations under the assumptions of hydrostatic pressure distribution and negligible vertical velocity. With the appropriate initial conditions and boundary conditions, these nonlinear equations can be solved numerically. Researchers have developed various numerical models to solve the SWEs over the past few decades. The Alternating Direction Implicit (ADI) scheme is one such method which is still widely used. This scheme was first proposed by Peaceman and Rachford (1955), after which Leendertse combined the ADI scheme and a staggered grid and developed a computational model for two-dimensional flows. The stability domain of this scheme is relatively large, and the balance between computational cost and accuracy is attractive (Leendertse and Gritton, 1971). However, Liang et al. (2006a) demonstrated that the ADI scheme is unable to predict trans-critical and supercritical flows. Two established approaches are suitable for resolving shocks, such as hydraulic jumps and bores, in the solution, *i.e.* shock-fitting and shock-capturing methods. Shock-capturing methods are preferable since they utilize a universal strategy over the whole domain without treating the shocks separately. As for most explicit schemes, time

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steps for shock-capturing schemes are restricted by the CFL condition.

Currently, the ADI scheme is still widely used in a lot of commercial and research software, such as Flood Modeller (www.floodmodeller.com) and Hec Ras (www.hec.usace.army.mil). Two widely-adopted numerical treatments are available for stabilising the ADI scheme in simulating rapid flows. One is to introduce artificial diffusion, and the other is to modify the Boussinesq coefficient. Physically, the Boussinesq coefficient quantifies the momentum effect of the non-uniform velocity distribution over the depth. Because it appears in the advective acceleration term in the SWEs, it can also be treated as a pure numerical parameter to tune the amount of fluid inertia to be considered in the simulation. Hence, its link to the vertical distribution of the velocity can be ignored. Instead, the Boussinesq coefficient is allowed to be freely tuned to eliminate the fictitious fluctuations and increase computational stability, just like the artificial viscosity coefficient. These treatments can partly mitigate the inherent instability of the ADI model when solving for trans-critical flows. However, their results may seriously deviate from the correct solutions to the original SWEs where the Boussinesq coefficient takes the physically-based values of around 1.0.

Although the effect of artificial viscosity on the computational results has been extensively studied, the influence of the Boussinesq coefficient is not well understood. This paper aims to demonstrate the impact of changing the Boussinesq coefficient when simulating rapid flows in various situations. The test cases first consider the instantaneous one-dimensional (1-D) and the two-dimensional (2-D) dam-break floods over frictionless flat beds, with the solutions predicted by a TVD-MacCormack model and experimental data used as references. Then, steady flow over frictional and irregular beds is simulated. Finally, the paper describes the applications of the 2-D ADI model to unsteady flood flows over an actual urban area in Glasgow, Scotland. In all the cases studied, the flow is either supercritical or nearly supercritical, and thus contains steep water level and velocity variations. This is because, when the flow is subcritical, the ADI model is capable of predicting the phenomena accurately with the correct value of β, which is close to unity. Considering that there is no need to take non-physical values of β when the flow is slow and smooth, this study focuses on situations with shocks present in the solutions.

2. Shallow water equations

The three-dimensional Reynolds-averaged continuity and Navier-Stokes equations are often integrated over the water column for analysing the flows confined in a thin layer. Utilising the assumption of the hydrostatic pressure distribution and the kinematic boundary condition of the free surface, the resulting equations are simplified into the SWEs. When neglecting the Coriolis, viscous and wind forces, the standard SWEs are expressed as:

$$\frac{\partial \eta}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{1}$$

$$\frac{\partial q_x}{\partial t} + \frac{\partial(\beta q_x^2/H)}{\partial x} + \frac{\partial(\beta q_x q_y/H)}{\partial y} = -gH\frac{\partial \eta}{\partial x} - \frac{g q_x\sqrt{q_x^2+q_y^2}}{H^2C^2} + \upsilon\left[2\frac{\partial^2 q_x}{\partial x^2} + \frac{\partial^2 q_x}{\partial y^2} + \frac{\partial^2 q_y}{\partial x \partial y}\right] \tag{2}$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial(\beta q_y^2/H)}{\partial y} + \frac{\partial(\beta q_x q_y/H)}{\partial x} = -gH\frac{\partial \eta}{\partial y} - \frac{g q_y\sqrt{q_x^2+q_y^2}}{H^2C^2} + \upsilon\left[\frac{\partial^2 q_y}{\partial x^2} + 2\frac{\partial^2 q_y}{\partial y^2} + \frac{\partial^2 q_x}{\partial x \partial y}\right] \tag{3}$$

where t is time; η represents the water surface elevation above the still water datum; q_x and q_y constitute the volumetric discharge components per unit width in the x and y directions, respectively; $H(= h + \eta)$ is the total water column depth, in which h is the depth below the still water datum; g is the gravitational acceleration; C represents the Chézy roughness coefficient which is determined from the Manning formula in this study; β is called the Boussinesq coefficient – a momentum-correction factor for the non-uniform vertical velocity profile; υ is the viscosity coefficient. For the dam-break flows, flash floods and other rapid flow phenomena considered in this paper, the fluid inertia, pressure force and bed friction are often the dominant factors influencing the flow, whereas the influence of viscosity is generally insignificant.

Assuming the flow is along the x-direction, then the Boussinesq coefficient can be defined as:

$$\beta = \frac{\int_{z_b}^{\eta} u^2 dz}{U^2 H} \tag{4}$$

where the flow velocity u is a function of the vertical coordinate z, and U is the depth-averaged velocity. This momentum correction factor depends on the velocity distribution over the depth, and should be always greater than unity. It increases with the increasing non-uniformity of the velocity distribution over the depth. For a logarithmic velocity profile, the momentum correction factor can be expressed as (Goldstein, 1938):

$$\beta = \left(1 + \frac{g}{C^2\kappa^2}\right) \tag{5}$$

where κ is von Kármán's constant. From the above equation, it can be seen that the value of β increases with the decrease of the Chézy coefficient, which signifies an increase in the bed roughness, vertical shearing and velocity non-uniformity. Alternatively, an assumed seventh-power law velocity profile leads to a constant value for β of 1.016. In many practical modelling studies, the momentum correction factor is simply set to unity, assuming a uniform velocity distribution. On the other hand, because β appears in the advective acceleration term in Eqs. (2) and (3), it can also be used to adjust the amount of fluid inertia to be considered in the computer simulations. For example, the advective acceleration can be completed ignored by taking β to be zero. Some researchers adjust the Boussinesq coefficient to improve the stability of the ADI-based models, following an idea similar to the adjustment of the viscosity coefficient in the numerical simulations. In such cases, the value of β should be treated as purely artificial, just as the artificial viscosity coefficient. Physically, the value of β should never be smaller than unity according to the Eq. (5). However, in the numerical modelling, the advective acceleration can be completed ignored by setting the value of β to be zero.

Eqs. (1)–(3) are not in a conservative formulation of the SWEs. As a set of hyperbolic equations, the SWEs admit discontinuities in their solutions. In order to maintain the correct motion of the shocks in numerical prediction, the conservative form of the SWEs should be deployed to ensure the exact mass and momentum conservation in the numerical discretization (Toro, 2001; Liang et al. 2006a).

3. Numerical methods

3.1. ADI scheme

The ADI scheme is widely used in practice to solve the SWEs. In the ADI scheme, each time step is divided into two half time steps. In the first half time step, the x-direction terms and derivatives are discretised using an implicit scheme, while an explicit scheme is used to approximate the values associated with the y-direction