Journal of Hydrology 557 (2018) 128–136

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

A three-dimensional model for quantification of the representative elementary volume of tortuosity in granular porous media

Ming Wu [a,b], Jianfeng Wu [a,*], Jichun Wu [a,*], Bill X. Hu [b]

[a] *Key Laboratory of Surficial Geochemistry, Ministry of Education, Department of Hydrosciences, School of Earth Sciences and Engineering, Nanjing University, Nanjing 210023, PR China*
[b] *Institute of Groundwater and Earth Sciences, Jinan University, Guangzhou 510632, PR China*

ARTICLE INFO

Article history:
Received 18 September 2017
Received in revised form 4 December 2017
Accepted 9 December 2017
Available online 11 December 2017
This manuscript was handled by Corrado Corradini, Editor-in-Chief, with the assistance of Philip Brunner, Associate Editor

Keywords:
Granular porous media
Tortuosity
Flow path
Regular tetrahedron
Representative elementary volume

ABSTRACT

For most of aquifers with abundant groundwater resource, quantifications of tortuosity and corresponding representative elementary volume (REV) are very essential to improve the understanding of groundwater and contaminant transfers in porous media. In this study, a mathematical model of tortuosity based on the three dimensional (3D) microstructure of regular tetrahedron (RTM) is proposed to quantify tortuosity and corresponding REV of granular porous media. The calculated tortuosity using the new 3D RTM model agrees well with the measured tortuosity in experiment, indicating that the new 3D microstructure model is more appropriate to precisely delineate the tortuosity of granular porous media. Afterward, the new model is utilized to quantify the tortuosity of heterogeneous translucent silica. Moreover, corresponding REV is estimated using a criterion of relative gradient error (ε_g^i). Results suggest minimum τ-REV sizes most distribute in 0.0–5.0 mm and the bound of cumulative frequency above 80% is larger than 3.00 mm. The REV scale of tortuosity has its own rationality and superiority over that estimated by two-dimensional (2D) tortuosity model, implying the proposed 3D tortuosity model of RTM is helpful for understanding the tortuosity of flow paths in granular porous media and corresponding REV estimation of tortuosity.

1. Introduction

As a very important component of freshwater resources, groundwater is indispensable to life on earth. Domestic, agricultural, industrial and ecosystem water consumption all rely on groundwater (Bakshevskaia and Pozdniakov, 2016; Cui et al., 2016; Liu et al., 2016; Wu et al., 2017a,b). Usually, abundant groundwater resources are stored in aquifer composed of granular porous media, like coastal and river bank aquifers. With the development of industry and agriculture, groundwater overexploitation caused by human beings become more and more serious, especially for the granular aquifers stored rich groundwater resources (Malov, 2016; Rezaei et al., 2016; Shoushtari et al., 2016; van Dijk et al., 2016; Zakari et al., 2016). In spite of the great increase of groundwater overexploitation, the granular aquifer also is contaminated by various contaminants released from human activities (Tye and Lapworth, 2016). The macroscopic transport of water and contaminant in granular porous media are related to the tortuosity which is the ratio of the actual length of flow path to the straight length along the macroscopic pressure gradient (Scheidegger, 1974; Sahimi, 1993; Koponen et al., 1996; Clennell, 1997; Yu and Cheng, 2002; Yu et al., 2003; Yun et al., 2006; Matyka et al., 2008). As a consequence, a quantitative understanding of tortuosity of granular porous media is crucial to improve the understanding of water and contaminants transport in porous media and designing associated remediation scheme. The flow path of water and contaminant transfer in granular porous media is very complex, which makes the tortuosity hard to be characterized (Carman, 1937; Carman, 1956; Kubik, 1986; Adler, 1992; Koponen et al., 1996; Lovell and Harvey, 1997; Yun et al., 2005; Yun et al., 2006, Cieszko, 2009). Tortuosity is a ratio which characterizes tortuous flow path of fluid diffusion and electrical conduction. The complexity of flow path in porous media leads to different transport phenomena and various definitions of tortuosity, such as diffusive, electrical and acoustic tortuosity definitions (Clennell, 1997; Matyka et al., 2008). The tortuosity of porous media is significant which related to groundwater transport and contaminants migration. Some experiments were conducted to measure tortuosity of porous media and obtain the experimental correlation equation between tortuosity and some empirical parameters (Wyllie and Gregory, 1955; Comiti and Renaud, 1989). To achieve the

---

* Corresponding authors.
E-mail addresses: jfwu@nju.edu.cn (J. Wu), jcwu@nju.edu.cn (J. Wu).

https://doi.org/10.1016/j.jhydrol.2017.12.030
0022-1694/© 2017 Elsevier B.V. All rights reserved.

mechanism of tortuosity of flow path in porous media, numerical methods (Koponen et al., 1996) and analytical models (Yun et al., 2005; Yun et al., 2006) were developed to determine tortuosity. Based on two dimensional (2D) porous media composed of cubic and spherical particles, Yu and Li (2004) developed simple models to delineate tortuosity of porous media. Yun et al. (2005) also proposed a tortuosity model of tortuosity based on 2D microstructures of regular triangle arrangement (RTA) and square pitch arrangement (SPA). To estimate the representative elementary volume (REV) (Brown and Hsieh, 2000; Zhang et al., 2000; Matthäi and Belayneh, 2004; Costanza-Robinson et al., 2011; Yoon and Dewers, 2013) of tortuosity for translucent granular porous media, a 2D model of tortuosity was proposed by Wu et al. (2017a,b,c,d) to calculate tortuosity and quantify the REV of tortuosity. However, most of previous models of tortuosity were idealized, of which the ability of determining tortuosity were not very suitable for realistic granular porous media. Yun et al. (2006) developed a three dimensional (3D) model to determine tortuosity of porous media composed of spherical particles, while the 3D microstructure of porous media used to derive the mathematical model of tortuosity is too idealized and can be regarded as a pseudo 3D microstructure. Therefore, previous models are only suitable for 2D porous media, while tortuosity of realistic 3D porous media is still needed to be explored and delineated further.

To overcome the limitations of previous tortuosity models for granular porous media, this study develops a new 3D tortuosity model based on regular tetrahedron microstructure (RTM) composed of spherical solid particles and pores among them. The analytical tortuosity model of flow path in 3D granular porous media is derived by mathematics. Experimental correlation equation for tortuosity in porous media composed of spherical particles is used to verity the tortuosity model proposed in this study. Moreover, tortuosity model is also compared against experiment data of tortuosity. Furthermore, an experiment is conducted to measure the tortuosity of granular porous media. Heterogeneous translucent silica is packed in a 2D sandbox and porosity is achieved using light transmission micro-tomography (LTM) technique (Wu et al., 2017a,b,c,d). Afterward, the tortuosity of experimental heterogeneous translucent silica is derived using the 3D tortuosity model of RTM. By the help of a criterion of relative gradient error (ε_g^i), The REV of tortuosity of the heterogeneous translucent silica is quantified and associated statistical assessment on REV sizes is conducted.

2. Geometry models for tortuosity of streamlines in 3D porous media

2.1. A new geometry model for tortuosity of streamlines in 3D porous media

Tortuosity is defined as the ratio of the actual length of tortuous flow path to the straight length of flow path along the pressure gradient (Scheidegger, 1974; Sahimi, 1993; Koponen et al., 1996; Clennell, 1997; Yu and Cheng, 2002; Yu et al., 2003; Yun et al., 2006; Matyka et al., 2008; Wu et al., 2017a,b):

$$\tau = \frac{L_a}{L_s} \tag{1}$$

where τ is tortuosity; L_a is the actual length of tortuous flow path; L_s is straight length along the pressure gradient for flow path.

In granular porous media, there are numerous complex flow path for water and contaminant transport. To calculate the tortuosity of flow path in granular porous media, the average tortuosity is derived using average method (Yun et al., 2005; Yun et al., 2006):

$$\tau = \frac{1}{N}\sum_i \tau_i \tag{2}$$

where N is the number of all possible tortuous flow paths in granular porous media; τ_i is the tortuosity of i^{th} flow path.

For 2D RTA (Fig. 1a), the ratio of gap length d between solid particles to median radius of solid particles $p_{RTA} = \frac{d}{R_v}$ is derived by Yun et al. (2005):

$$p_{RTA} = \frac{d}{R_v} = \sqrt{\frac{2\pi}{\sqrt{3}(1-n)}} - 2 \tag{3}$$

where d is the length of the gaps between solid particles in granular porous media; R_v is the radius of solid particles; n is porosity.

The ratio of gap length d between solid particles to median radius of solid particles $p_{SPA} = \frac{d}{R_v}$ for 2D SPA (Fig. 1b) is calculated as (Yun et al., 2005):

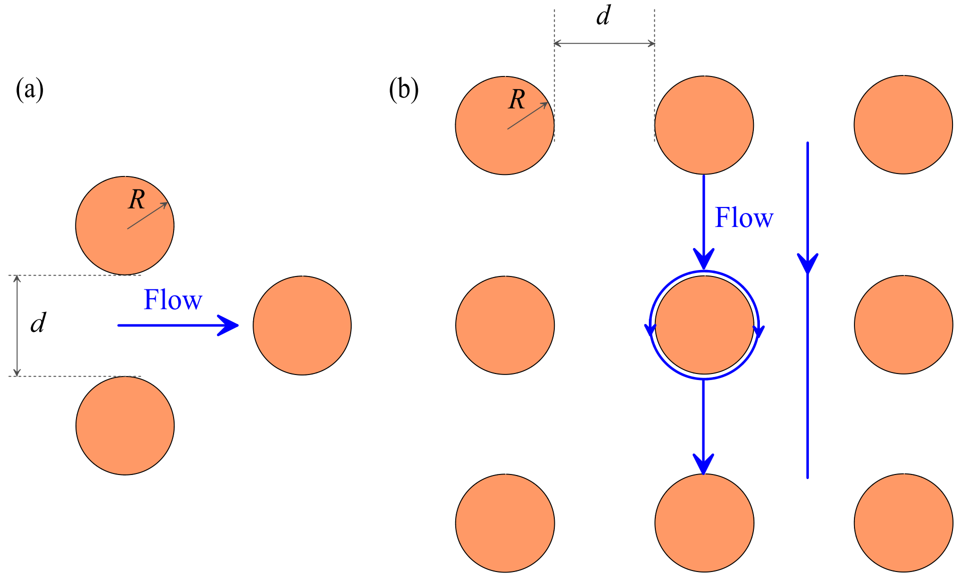

Fig. 1. (a) 2D microstructure of RTA (Yun et al., 2005; Wu et al., 2017a); and (b) 2D microstructure of SPA (Yun et al., 2005).